VELAMMAL INSTITUTE OF TECHNOLOGY

PANCHETTI.

DEPARTMENT OF MECHANICAL ENGINEERING

ME6502- HEAT AND MASS TRANSFER

ME6502- HEAT AND MASS TRANSFER

UNIT- I

CONDUCTION

General Differential equation of Heat Conduction– Cartesian and Polar Coordinates – One dimensional Steady State Heat Conduction – plane and Composite Systems – Conduction with Internal Heat Generation – Extended Surfaces – Unsteady Heat Conduction – Lumped Analysis – Semi Infinite and Infinite Solids – Use of Heisler's charts

UNIT II

CONVECTION

Free and Forced Convection - Hydrodynamic and Thermal Boundary Layer. Free and Forced Convection during external flow over Plates and Cylinders and Internal flow through tubes

UNIT III

PHASE CHANGE HEAT TRANSFER AND HEAT EXCHANGERS

Nusselt's theory of condensation - Regimes of Pool boiling and Flow boiling. Correlations in boiling and condensation. Heat Exchanger Types - Overall Heat Transfer Coefficient – Fouling Factors - Analysis – LMTD method - NTU method.

UNIT IV

RADIATION

Black Body Radiation – Grey body radiation - Shape Factor – Electrical Analogy – Radiation Shields. Radiation through gases.

UNIT V

MASS TRANSFER

Basic Concepts – Diffusion Mass Transfer – Fick's Law of Diffusion – Steady state Molecular Diffusion– Convective Mass Transfer – Momentum, Heat and Mass Transfer Analogy – Convective MassTransfer Correlations.

UNIT- I CONDUCTION

PART-A

TWO MARKS QUESTIONS AND ANSWERS:

1. State Fourier's law of heat conduction. (May/June 2013, Nov/Dec 2013, April/May 2011)

This Fourier equation is used to find out the conduction heat transfer. According to this equation, heat transfer is directly proportional to surface area and temperature gradient. It is indirectly proportional to the thickness of the slab.

$$Q \propto \frac{A\Delta T}{L}$$

$$Q = \frac{-kA\Delta T}{L}$$

$$Q = \frac{-kA\ (T_2 - T_1)}{L}$$

$$Q = \frac{kA\ (T_1 - T_2)}{L}$$

2. Define fin efficiency and fin effectiveness. (May/June 2013, Nov/Dec 2010).

η_{fin} = Efficiency of fin = $\frac{Q}{Q_{max}}$

$$\eta_{fin} = \frac{\text{Heatlostbyfin}}{\text{Heatlossbythefins,ifwholesurfaceofthe finismaintainedatroot (basetemperature)}}$$

$$\eta_{fin} = \frac{\tan hmL}{mL}$$

Where m = $\sqrt{\frac{hP}{kA_c}}$

P = Perimeter

A_c = Cross sectional area

Efficiency of fin is defined as ratio of actual heat transfer from fin to the max. Heat transfer from fin.

ε = Effectiveness of fin (or) Area weighted fin efficiency

$$= \frac{Q_{withfin}}{Q_{withoutfin}}$$

$$= \frac{Q_{withfin}}{hA\Delta T}$$

Where A = Surface area

h = Convective heat transfer coefficient (film heat transfer coefficient)

Effectiveness of fin is defined as the ratio of heat transfer with fin to the heat transfer without fin on the same cross sectional area.

3. What is lumped system analysis? When is it used? (May/June 2013, April/May 2011,Nov/Dec 2010)

When Bi ≤ 0.1, we use lumped capacity analysis. That is, the internal resistance is negligible when compared to surface resistance. Lumped capacity type of analysis assumes a uniform temperature distribution throughout the solid body since internal conduction resistance is very less when compared with surface convection resistance.

Lumped capacity analysis yield good results for many practical cases

4. Write the three dimensional heat transfer Poisson's and Laplace equations in Cartesian co-ordinates. (May/June-2012)

When the temperature is not varying with respect to time, then the conduction is called as steady state conduction.

i.e., $\frac{\partial T}{\partial \tau} = 0$

Then the general equation becomes Poisson's equation as

$\nabla^2 T + \frac{q_g}{k} = 0$

Where $\nabla^2 = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$

When the conduction is steady state condition, (i.e.,$\partial T/\partial \tau = 0$) and there is no heat generation, the general equation becomes Laplace equation as

$\nabla^2 T = 0$

Where $\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$

5. A 3 mm wire of thermal conductivity 19 W/mK at a steady heat generation of 500 MW/m³. Determine the centre temperature if the outside temperature is maintained at 25℃. h = 4500W/m²K (May/June 2012)

Given:

Radius of wire, R = 3mm = 3×10^{-3} m

Thermal conductivity, k = 19 W/mK

Heat generation = 500 MW/m^3

Outside temperature = 25℃ =298 K

To Find

Centre temperature

Solution

$T_w = T_\infty + \frac{qR}{2h}$

$= 25 + \frac{500\times10^6\times0.003}{2\times4500}$

$= 191.66$℃

$T_{r=0} = T_w + \frac{500\times10^6}{4\times19}(0.003^2 - 0)$

$=250.87$℃

6. What are the two mechanisms of heat conduction in solids?(Nov/Dec 2011)

(a) Conduction

(b) Convection

7. What is the purpose of attaching fins to a surface? What are the different types of fin profiles?(Nov/Dec 2011)

The main purpose of attaching fins is to increase the heat transfer rate.

The fin profiles are

- Concave profile
- Convex profile
- Parabolic profile

8. In what medium, the lumped system analysis is more likely to be applicable? Aluminium or wood? Why?(Nov/Dec 2011)

Lumped system analysis is more likely applicable to Aluminium because for Aluminium the internal resistance $\left(\frac{1}{K_A}\right)$ is negligible as compared with wood.

9. What is heat generation in solids? Give examples. (April/May 2011)

In many practical cases, there is a heat generation within thesystem.

Examples:

(a) Electric coils
(b) Resistance heater
(c) Nuclear reactor.

In electric coil and resistanceHeater, heat is generated due to electric current flowing in the wire.

10. Discuss the mechanism of heat conduction in solids. (May/June 2009)

In solids, heat is conducted by following the mechanisms

- By lattice vibration
- By transport of free electrons

11. What is the physical meaning of Fourier number? (May/June 2009)

Fourier number $F_o = \frac{\alpha\tau}{L_i^2}$

It signifies the degree of penetration of heating or cooling effect through a solid.

12. A temperature difference of 500℃ is applied across a fire-clay brick, 10cm thick having a thermal conductivity of 1 W/mK. Find the heat transfer rate per unit area. (Apr/May2008)

As per Fourier's law of heat conduction

$$\frac{Q}{A} = K\frac{dT}{dx}$$

$= 1 \times \frac{500}{0.1}$ = 5000 W/m℃

13. Write the general 3-D heat conduction equation in cylindrical co-ordinates.(Apr/May2008)

$$\left(\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r} + \frac{1}{r^2}\frac{\partial^2 t}{\partial \emptyset^2} + \frac{\partial^2 t}{\partial z^2}\right) + \frac{q_g}{k} = \frac{1}{\alpha}\frac{\partial t}{\partial \tau}$$

14. Biot number is the ratio between .... and .... (Apr/May 2008).

Biot number is the ratio between internal (conduction) resistance and surface (convection) resistance

15. What is the main advantage of parabolic fins? (Nov/Dec 2007)

A fin of parabolic profile is very effective in the sense that it dissipates the maximum amount of heat at minimum material cost.

16. What is sensitivity of a thermocouple? (Nov/Dec 2007)

The time required by a thermocouple to reach 63.2% of the value of initial temperature difference is called its sensitivity.

17. Define critical radius of insulation. (Nov/Dec 2007)

Critical radius of insulation is defined as the radius of insulation at which the heat loss is maximum.

18. Mention the importance of Biot number. (Nov/Dec 2007)

Biot number is a non-dimensional number used to test the validity of lumped heat capacity approach.

20. What is use of Heisler's chart? (May/June 2007)

Heisler's charts are used to solve problems – Transient heat conduction insolids with finite conduction and convective resistances. i.e $0 < Bi < 100$.

21. Define heat transfer.

Heat transfer can be defined as the transmission of energy from one regionto another due to temperature difference.

22. What are the modes of heat transfer?

1. Conduction.
2. Convection.
3. Radiation

23. What is conduction?

Heat conduction is a mechanism of heat transfer from a region of hightemperature to a region of low temperature with in a medium (solid, liquidor gases) or different medium in directly physical contact. In conduction,energy exchange takes place by the kinematic motion or direct

24. Define Convection.

Convection is a process of heat transfer that will occur between a solidsurface and a fluid medium when they are at different temperatures Convection is possible only in the presence of fluid medium.

25. Define Radiation.

The heat transfer from one body to another without any transmitting mediumis known as radiation. It is an electromagnetic wave phenomenon.

26. Define Thermal conductivity.

Thermal conductivity is defined as the ability of a substance to conduct heat.

27. List down the three types of boundary conditions.

1. Prescribed temperature
2. Prescribed heat flux
3. Convection boundary conditions

28. Explain about Poisson's equation.

When the temperature is not varying with respect to time, then theconduction is called as steady state conduction.

29. What is critical radius of insulation?

Critical radius (rc): it is defined as outer radius of insulation for which theheat transfer rate is maximum.

Critical thickness: it is defined as the thickness of insulation for which theheat transfer rate is maximum.

30. What are the factors affect thermal conductivity?

1. Material structure. 2. Moisture content. 3. Density of material. 4. Pressure and temperature.

31. What is super insulation and give its application.

Super insulation is a process which is used to keep the cryogenic liquids atvery low temperature. The super insulation consists of multiple layers ofhighly reflective material separated by insulating spacers. The entire systemis evacuated to minimize air conduction.

32. Give some examples of heat generation application in heat conduction.

1. Fuel rod – nuclear reactor. 2. Electrical conductor. 3. Chemical and combustion process. 4. Drying and setting of concrete.

33. Define overall heat transfer co-efficient.

The overall heat transfer is defined as amount of transmitted per unit areaper unit time per degree temperature difference between the bulk fluids oneach side of the metal. it is denoted by 'U'.Heat transfer, $Q = UA\,\Delta T$.

34. Define fins (or) extended surfaces.

It is possible to increase the heat transfer rate by increasing the surface of heat transfer. The surfaces used for increasing heat transfer are called extended surfaces or sometimes known as fins.

35. What is meant by steady stale heat conduction?

If the temperature of a body does not vary with time, it is said to be in asteady state and that type of conduction is known as steady state heatconduction.

36. What is meant by Transient heat conduction or unsteady stateconduction?

If the temperature of a body varies with time, it is said to be in a transientstate and that type of conduction is known as transient heat conduction orunsteady state conduction.

37. What is Periodic heat flow?

In periodic heat flow, the temperature varies on a regular basis.

Examples:

1. Cylinder of an IC engine.
2. Surface of earth during a period of 24 hours.

What is non periodic heat flow?

In non periodic heat flow, the temperature at any point within the system varies non linearly with time.

Examples:

1. Heating of an ingot in a furnace.
2. Cooling of bars.

38. What is meant by Newtonian heating or cooling process?

The process in which the internal resistance is assumed as negligible incomparison with its surface resistance is known as Newtonian heating orcooling process.

39. What is meant by Lumped heat analysis?

In a Newtonian heating or cooling process the temperature throughout thesolid is considered to be uniform at a given time. Such an analysis is calledLumped heat capacity analysis.

40. What is meant by Semi-infinite solids?

In a semi infinite solid, at any instant of time, there is always a point wherethe effect of heating or cooling at one of its boundaries is not felt at all. Atthis point the temperature remains unchanged. In semi infinite solids, thebiot number value is ∞.

41. What is meant by infinite solid?

A solid which extends itself infinitely in all directions of space is known asinfinite solid. In infinite solids, the biot number value is in between 0.1 and100.

42. Explain the significance of Fourier number.

It is defined as the ratio of characteristic body dimension to temperaturewave penetration depth in time.It signifies the degree of penetration of heating or cooling effect of a solid.

43. What are the factors affecting the thermal conductivity?

1. Moisture. 2. Density of material. 3. Pressure. 4. Temperature 5. Structure of material.

44. Explain the significance of thermal diffusivity.

The physical significance of thermal diffusivity is that it tells us how fastheat is propagated or it diffuses through a material during changes oftemperature with time.

45. Write down the equation for conduction of heat through a slab or plane wall.

Heat transfer $Q = \frac{\Delta T_{overall}}{R}$ Where $\Delta T = T_1 - T_2$

$R = \frac{L}{KA}$ - Thermal resistance of slab

L = Thickness of slab, K = Thermal conductivity of slab, A = Area

46. Write down the equation for conduction of heat through a hollow cylinder.

Heat transfer $Q = \frac{\Delta T_{overall}}{R}$ Where, $\Delta T = T_1 - T_2$

$R = \frac{1}{2\pi LK} \text{in} \left[\frac{r_2}{r_1}\right]$ thermal resistance of slab

L – Length of cylinder, K – Thermal conductivity, r_2 – Outer radius , r_1 – inner radius

47. State Newton's law of cooling or convection law.

Heat transfer by convection is given by Newton's law of cooling

$Q = hA (T_s - T_\infty)$

Where

A – Area exposed to heat transfer in m^2, h - heat transfer coefficient in W/m^2K

T_s – Temperature of the surface in K, T_∞ - Temperature of the fluid in K.

48. Write down the general equation for one dimensional steady state heat transfer in slab or plane wall with and without heat generation.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\infty} \quad \frac{\partial T}{\partial t} \quad \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{K} = \frac{1}{\alpha} \quad \frac{\partial T}{\partial t}$$

49. Write down the equation for heat transfer through composite pipes or cylinder.

Heat transfer $Q = \frac{\Delta T_{overall}}{R}$, Where , $\Delta T = T_a - T_b$,

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{In\left[\frac{r_2}{r_1}\right]}{K_1} + \frac{In\left[\frac{r_1}{r_2}\right]L_2}{K_2} + \frac{1}{h_b r_3}\right].$$

50. What is critical radius of insulation (or) critical thickness? [Nov/Dec-2014]

Critical radius = r_c Critical thickness $t_c = r_c - r_1$

Addition of insulating material on a surface does not reduce the amount of heat transfer rate always. In fact under certain circumstances it actually increases the heat loss up to certain thickness of insulation. The radius of insulation for which the heat transfer is maximum is called critical radius of insulation, and the corresponding thickness is called critical thickness.

51. State the applications of fins.

The main applications of fins are

1. Cooling of electronic components
2. Cooling of motor cycle engines.
3. Cooling of transformers
4. Cooling of small capacity compressors

Part –B

1. At a certain instant of time, temperature distribution in a long cylindrical tube is T = 800 +100 r- 5000 r^2 where, T is in °C and r in mm. The inner and outer radii of the tube are respectively 30 cm and 50 cm. the tube material has a thermal conductivity of 58 W/m.K and a thermal diffusivity of 0.004 m^2/hr. Determine the rate of heat flow at inside and outside surfaces per unit length, rate of heat storage per unit length and rate of change of temperature at inner and outer surfaces. (May/June-2013)

Given: In cylindrical tube,

$$T = 800 + 1000\,r - 5000\,r^2$$

Inner radius, r_1 = 30 cm = 30 x 10^{-2} m

Outer radius, r_2 = 50 cm = 50 x 10^{-2} m

Thermal conductivity, K = 58 W/mK

Thermal diffusivity, α = 0.004 m^2/hr

$$= \; = \frac{0.004}{3600} = 1.11 \times 10^{-6} \; m^2 / s$$

To find:

1. Rate of heat flow at inside and outside surfaces per unit length.
2. Rate of heat storage per unit length.
3. Rate of change of temperature at inner and outer surfaces.

Solution:

1. Rate of heat flow at inside surfaces per unit length.

$$Q_{in} = -KA_i \left(\frac{dT}{dr} \right)_{r_i = 0.3}$$

$$Q_{in} = -58 \times 2\pi \times (0.3) \times 1 \times \left[\frac{d\left(800 + 1000r + 5000r^2\right)}{dr} \right]_{r_i = 0.3}$$

$$Q_{in} = -109.33\left[-2000\right] = 21.86 \times 10^4 W$$

Rate of heat flow at outside surfaces per unit length, Q_{out}

$$= -K \; A_0 \left(\frac{dT}{dr} \right)_{r_0 = 0.5}$$

$$Q_{out} = -58 \times 3.14 \times \left[\frac{d\left(800 + 1000r + 5000r^2\right)}{} \right]_{r_0 = 0.5}$$

$$= -58 \times 3.14 \times \left[-4000\right]$$

$$Q_{out} = 72.84 \times 10^4 W$$

Rate of heat storage per unit length.

$$\therefore Q_{stored} = Q_{in} - Q_{out}$$

$$= (21.86 - 72.84) \times 10^4$$

$$Q_{stored} = -50.98 \times 10^4 W$$

$$T = 800 + 1000r + 5000r^2$$

$$\frac{dT}{dr} = 1000 - 10000r$$

$$\frac{d^2T}{dr^2} = -10000$$

Rate of change of temperature at inner surfaces, at r_i = 0.3 m

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = \frac{1}{\alpha}.\frac{dT}{dt}$$

$$-10000 + \frac{1}{0.3}(1000 - 10000 \times 0.3) = \frac{1}{1.11 \times 10^{-6}}\left(\frac{dT}{dt}\right)_{r_i = 0.3}$$

$$\left(\frac{dT}{dt}\right)_{r_i = 0.3} = 0.01851^oC / s$$

Rate of change of temperature at outersurfaces,

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = \frac{1}{\alpha}.\left(\frac{dT}{dt}\right)_{r_o = 0.5}$$

$$-10000 + \frac{1}{0.5}(1000 - 5000 \times 2 \times 0.5) = \frac{1}{1.11 \times 10^{-6}}\left(\frac{dT}{dt}\right)_{r_o = 0.5}$$

$$\left(\frac{dT}{dt}\right)_{r_o = 0.5} = -0.02^oC / s$$

2. Circumferential rectangular fins of 140mm wide and 5mm thick are fitted on a 200mm diameter tube. The fin base temperature is 170^{o}C and the ambient temperature and the ambient temperature is 25^{o}C. Estimate fin efficiency and heat loss per fin.

Take: Thermal conductivity, k = 220 W/mK.

Heat transfer co-efficient, h = 140 W/m^{2}K (May/June-2013)

Given:

Wide, L =140 mm = 0.140 m

Thickness, t = 5 mm = 0.005 m

Diameter, d = 200 mm, r = 100 mm = 0.1 m

Fin base temperature, T_b = 170°C + 273 = 443 K

Ambient temperature T_∞= 25°C + 273 = 298 K

Thermal conductivity, k = 220 W/mK.

Heat transfer co-efficient, h = 140 W/m^2K.

To find:

1. Fin efficiency, η
2. Heat loss, Q

Solution:

A rectangular fin is long and wide. So, heat loss is calculated by using fin efficiency curves. [From HMT data book page no. 50 sixth edition]

Corrected length, $L_c = L + \frac{t}{2}$

$= 0.140 + \frac{0.005}{2}$

$L_c = 0.1425$ m

$r_{2c} = r_1 + L_c$

= 0.100+ 0.1425

$r_{2c} = 0.245$ m

$A_s = 2\pi[r_{2c}^2 - r_1^2]$

$A_s = 2\pi[(0.2425)^2 - (0.100)^2]$

$A_s = 0.30650\ m^2$

$A_m = t[r_{2c} - r_1]$

$A_m = 7.125 \times 10^{-4\ m2Q\ =}$

From graph, WKT

$X_{axis} = L_c^{1.5} [\frac{h}{kA_m}]^{0.5}$

$X_{axis} = 1.60$

Curve = r_{2C}/r_1 = 2.425

By using these values we found that the efficiency of the fin is 28%. (From the graph) Pg: No:50

Heat transfer Q = 0.28 x 0.30650 x 140 x (443-298) = 1742.99W.

Result:

1. **Fin efficiency = 28%**
2. **Heat loss Q = 1742.99 W**

3. A furnace wall is made up of three layer thickness 25cm, 10cm, and 15cm with thermal conductivities of 1.65w/mk and 9.2 w/mk respectively .the inside is exposed to the gasses at 1250^0c with is convection coefficient of 25 $w/m^{20}c$ and inside surface of 1100^0c ,the outside surface is exposed to the air at 25^0c with convection coefficient of 12 w/m^2K .determine (1)the unknown thermal conductivity (2) THE overall heat transfer coefficient (3) ALL surface temperature [May/June-12]

Given data :

Thickness $L_1=25*10^{-2}$m
thermal conductivity, K1=1.65 w/mk
$L_2=10*10^{-2}$m
$K_2=?$
$L_3=15*10^{-2}$m
$K_3=9.2$ w/mk
$Ta=1250^0C$ =1523 K, $T_1=1100^0C$ =1373 k ;$T_b=25^0c$ =298K
$h_a=25\ w/m^{20}c$; $h_b=12\ w/m^2k$

To find :

(a) Unknown thermal conductivity ,K_2
(b) Overall heat transfer coefficient ,U
(c) All the surface temperature (T_2,T_3,T_4)

Solution :

Q=25*1*(1250-1100)=3750W

$Q=T_a-T_b/1/A\ [1/h_a+l_1/k_1+l_2/k_2+l_3/k_3+1/h_p]$

Q conducted =Qconvected

$3750=1250-25/1/1[1/25+25*10^{-2}/1.65+10*10^{-2}/K_2+15*10^{-2}/9.2+1/2]$

$0.2912+10*10^{-2}/K_2=0.3266$

$K_2=2.82$ w/mk

$Q = UA\ (T_a-T_b)$

$Q=T_1-T_2/L_1/KA$

$3750= UA\ (T_a-T_b)$

$3750=1100-T_2/25*10^{-2}/1.65*1$

$3750= U1\ (T_a-T_b)$

$T_2=531.71^0c$ or 804.81K

$U=3.061$ w/m^{2}k

$Q=T_2-T_3/L_1/k_2A$

$Q=T_3-T_4/L_1/K^2A$

$3750=531.81-T_3/10*10^{-2}/2.82*1$

$3750=398.83 -T_4/15*10^{-2}/9.2*1$

$T_3=398.83^0C$

$T_4=337.68^0c$

4. Pin fins are provided to increase the heat transfer rate from hot surface .which of the following arrange will given higher heat transfer rate ?(1) 6 fins of 10 cm length (2) 12 fins of 5cm length .take K of fin material =200 w/mk and h =20w/m^{20}c cross sectional area of the fins =2cm^2,perimeter of fin =4cm ,find the base temperature =230^{0}c, surrounding air temperature =300^{0}c [May /June 12]

Given data :

Case (1) No. of fin ,S =6 ; length ,L $=10*10^{-2}$m

CASE (2) no. of fin , S=12, length ,L$=5*10^{-2}$m

Thermal conductivity , K=200 w/mk

Heat transfer coefficient , h = 20 w/m^{20}c

Cross sectional area of fin , A= 2cm^2=$2(1*10^2)^2=20*10^{-4}$m^2

Perimeter of fin , P $=4*10^{-2}$m

Fin base temperature , $T_b=230^0c$ =503 K

Air temperature , $T\infty=300^0c$ =303K

To find :

Higher heat transfer rate (Q)

Solution :

Assume short fin [end insulated]

Case(1):

$Q=(hpKA)^{0.5}(T_b-T_a).\tanh(mL)$

$M=\sqrt{hp/KA}= \sqrt{20*10^{-2}*4/200*2*10^{-4}} =4.472\ m^{-1}$

$Q=(20*4*10^{-2}*200*10^{-4}*2)^{0.5}(503-303)\ \tanh\ (4.472*10*10^{-2})$

Q=15.01 w/fin

Heat transfer for 6fins =15.01*6 =90.07 W

Case (2) :

$Q= (20*10^{-2}*4*200*2*10^{-4})^{0.5}(503-303)+ 0(4.472*10*10^{-2})$

Q= 7.86 w/fin

Heat transfer rate for 12 fins =7.86*12 =94.42 W

Result:

The higher heat transfer, Q=94.42 W for no. of fins =12

5.A composite wall consists of 2.5 cm thick copper plate,a 3.2 cm layer of asbestos insulation and 5cm layer fiber plate .thermal conductivities off the material are respectively 355,0.110 and 0.0489 w/mk. the temperature difference across the composite wall is 560^{0}c the side and ^{0}c on the other side. find the heat flow through the wall per unit area and the interface temp .between asbestos and fiber plate.[Nov/Dec-12]

Given data:

$L_1=2.5*10^{-2}m$ $K_1=355$ w/mk

$L_2=3.2 *10^{-2}m$ $K_2=0.11$ w/mk

$L_3=5*10^{-2}m$ $K_3= 0.0489$ w/mk

Temperature,$T_1= 560^0c =833k$; $T_4= 0^0c =273\ K$

To find:

(a) heat flow per unit area ,Q/A
(b) interface temperature between asbestos and fibre plate ,T_3

Solution:

(a) heat transfer ,$Q=T_1-T_4/1/A\ [1/h_a+l_1/k_1+l_2/k_2+l_3/k_3+1/h_p]$
h_a and h_b not given .so neglected it .

$Q/A = 833-273/2.5*10^{-2}/355+3.2*10^{-2}/0.11+5*10^{-2}/0.0489$ c=426.35 w/m^2

$Q/A = T_1-T_2/L_1/k_1$

$426.35 = 560 - T_2/2.5*10^{-2}/355 = 559.95\ ^0c$

$Q/A = T_2-T_3/L_2/k_2$

$426.35 = 559.95-T_3/3.2*10^{-2}/0.11 = 435.9^0c$

Result:

(a) Q/A = 426.35 w/m^2 (b) $T_3 = 435.9\ ^0c$

6. The cylinder of a 2-stroke SI engine is constructed of aluminum alloy (K=186 w/mk).The height and outside diameter of the cylinder are respectively 15cm and 5cm.understand operating condition ,the outer surface the cylinder is at500k an is exposed to the ambient air at 3000K ,with a convention heat transfer coefficient of50 w/m^{2}K equally spaced annular fins are attached with cylinder to increase the heat transfer .there are five such fins with uniform thickness ,t=6mm and the length ,l=20mm. calculate the increase in heat transfer due to the addition fins [Nov/Dec-11]

Given data :

Thermal conductivity ,K= 186 w/mk

Length of the cylinder ,$L_{cy}=15*10^{-2}$m

Cylinder diameter ,d= $5*10^{-2}$ m

Ambient temperature ,T∞=300k

Cylinder surface temperature ,T_b =500 K

Heat transfer coefficient ,h= 50 w/m^{2}k

Number of fin =5

Fin thickness ,t=$6*10^{-3}$m ; fin length ,l_f= $20*10^{-3}$m

To find :

Increase in heat transfer due to addition of fins

Solution :

Fin length is a 20mm .so it is treated as short film

Heat transfer , $Q=(hpKA)^{0.5}(T_b-T_a).\tanh(mL_f)$

Perimeter ,p=2*($L_{cylinder}$+t) =2*(15 *10^{-2}+6*$10^{-3)}$

=0.312m

M=√hp/KA = √50*0.312/186*9*10^{-4}

m=9.563

Q=(50*0.312 *186*9*10^{-4})$^{0.5}$(500-300) tan h (9.653 *20*10^{-3})

Q=61.63W

Heat transfer /fin = 61.63 W

Heat transfer for five fin Q_1= 61.63 *5=308.16 W

Heat transfer for unfined surface [convection]

Q_2= hAΔT= h($\pi d L_{cy}$-5*t*L_f)(T_b-T_a)

=50(π*5*10^{-2}*15*10^{-2}-5*6*10^{-3}*20*10^{-3})(500-300)

=229.62W

Total heat transfer ,Q_3= Q_1+Q

=308.11+229.67 =537.78 W

Heat transfer without fin Q=hAΔT

=(π*5*10^{-2}*1*10^{-2})(500-300) =235.61 W

Increase in heat transfer due to addition of fin Q_3-Q =537.73-235.61 =302.17 W

7.A cold storage room has walls made of 23cm of bricks on the outsie,8cm of plastic foam and finally 1.5cm of wood on the inside .the outside and inside air temperature are22 and-2 respectively. the inside and outside heat transfer coefficient are respectively 29 and 12 w/m²k .the thermal conductivities of brick ,foam and wood are 0.98,0.02 and0.12 w/mk respectively .if the total wall area is 90m/t determine the rate of heat removal by refrigerator and the temperature of the inside surface of the brick [April/May-11]

Given data :

$L_{`1}$=23*10^{-2}m T_a= 22^0c =29

L_2=8*10^{-2}m T_b=-2^0c =271 K

L_3=1.5*10^{-3}m

Heat transfer coefficient ,h_a= 29 w/m^2k ,h_b12 wm^2k

Thermal conductivity ,K_1=0.98 w/mk ; K_2= 0.02 w/mk ;K_3= 0.12 w/mk

Area , A = 90m ^{2}

To find :

(a) Q (b) T1

Solution :

(a) Heat transfer , Q=T_a-T_b/1/A [1/h_a+l_1/k_1+l_2/k_2+l_3/k_3+1/h_p]

= 295-271 /1/90 [23*10^{-2}/0.98 +1.5*10^{-2}/0.12 +8*10^{-2}/0.02 +1/12]

24/1/90(4.473) =482.41 W

(b) Q=T_a-T_1/L_1/K_1A
428.41 =295 –T_1 /23*10^{-2}0.98*90
T_1 =293.74 K

Result :

Heat transfer, Q =482.71 W
Interface temperature, T1= 293.74 K

8.A steel rod of diameter 112mm and 60mm long with insulated end that has a thermal conductivity of 32w/m^{0}c is to be used as a spine .it is expressed to surrounding with a temperature at 60^{0}c and heat transfer coefficient of 55w/m^2 .the temperature the base of the fin is 95^{0}c .calculate the fin efficiency ,the temperature at the edge of the spine and the heat dissipation[Nov/Dec 10]

Given data :

Steel rod diameter ,d=12 *10^{-3}m

Length ,L=60*10^{-3}m ,thermal conductivity ,K =32 w/m ^{0}c ,surrounding temperature ,T_α= 60^{0}c =333K

Heat transfer coefficient ,h= 55 w/m^{2} ^{0}c ,base temperature of fin ,T_b = 95 ^{0}c =368K

To find :

(a)Fin efficiency η_{fin} (b)temperature at the edge of the spine ,(c)T heat dissipation ,Q

Solution :

(a) assume short fin (end insulated)

η_{fin} =tan h mL/mL ; m =√hp /KA

perimeter ,P =πd =3.14 *12*10^{-3} =0.0376m

area ,A =π/d^2= 3.14/4 $(12*10^{-3})^2$ =1.13*$10^{-4}$$m^2$

m =√hp /KA =√55*0.0376 /32 *1.13 *10^{-4} =23.91 m^{-1}

η_{fin} =tan h (23.91 *60*10^{-3})/ (23.91 *60*10^{-3}) =62.21 %

(b) temperature at the edge of the spine

T-T_a/T_b-T_a =cosh m(L-X)/cosh (mL)

T-333/368-333 = cosh (23.91-23.91)/cos h (23.91-60*10^{-3})

T =333K

(c) heat dissipation ,Q= Q=$(hpKA)^{0.5}$(T_b-T_a).tanh(mL)

$(55*0.0376*32*1.13 *10^{-4})^{0.5}$(368-333)tanh (23.91*60*$10^{-3}$)

Q =2.70 W

9. a) Two slabs each of 120mm thick have thermal conductivities of 14 w/m and 210 w/m .These are placed in contact but due to roughness only 30 of area placed in contact and gap in the remaining area is 0.025mm thick and is filled with air .If the temperature of the face of the hot surface is at 220 and the outside surface of the other slab is at 30 ,calculate the heat flow through the composite system .Assume that conductivity of the air is 0.032 and the half of the contact (of the contact area)is due to either metal[Nov/Dec 10]

Given data:

L_a =120mm =0.12 m ,L_{A1}=0.025 mm =0.000025m ,L_c=0.025 mm =0.000025 m=B_1

K_A=K_{A1}=14.3 w/m^0c ;K_B=K_{B1}=210 /m^0c ;K_c=0.032 w/m^0c

T_1=220^0c ;T_2=30^0c

To find heat flow through the composite system

R_{th}–A =L_A/$K_A$$A_A$= 0.12 /14.5 *1 = 0.00828 ^{0}c /w

$=L_B/K_BA_B= 0.12 /210*1 =0.0057\ ^0c/w$

$1/R_{eq}= 1/R_{A1} +1/R_c+1/R_{B1}$

$=14.5 *0.15 /0.000025 +0.0032*0.7/0.000025 +210*0.15/0.000025$

$R_{eq}=7.42 *10^{-7}$ c/w

R_{th}-total = R_{th}- A +R_{eq}+ R_{th}–B =0.00828+0.00057 +7.42 *10^{-7}=0.00885^{0}c/w

Q= ΔT/ R_{th}-total =220-30/0.00855 =2149 W =21.49 KW

10. A 60 mm thick large steel plate [K=42.6w/m^{0}c,X=0.043m^2/h] initially at 440^{0}c is suddenly exposed on the both side to an ambient with convection heat transfer coefficient 235w/m^{20}c and temperature inside the plate 15mm from the mid plane after 4.3minutes [Nov/Dec 10]

Givendata :

Thickness of the steel plate, L= 60*10^{-3}m

Thermalconductivity, K= 42.6 w/m^{0}c

Thermaldiffusivity, α= 0.043 m^2 /h = 0.043/3600 =1.94 *10^{-5} m^2/s

Initialtemperature, T1 =440^{0}c =713 K

Heat transfer coefficient, h=235 w/m^{0}c

Distance, X =15mm =15810^{-3}time ,t= 4.3 min =258 seconds

Tofind :

(a) Centre line temperature ,T_0
b) Temperature inside the plate 15mm from the mid plane ,T_x

Solution :

A) characteristics length ,L_c=L/2 =60*10^{-3}/2 =0.03

biot number , B_i= hL_c/k =235 *0.03/42.6=0.165

0.1<B <100 ,so it is infinite solid type

For infinite plane (mid plane)

Fourier number =$\alpha t/L^2_c$ =1.194 *10^{-5}*258/$(0.03)^2$ =3.422

y-axis = $T_0-T_\alpha/T_i-T_\alpha$ = 0.63

T_0 -323 /713 -323 = 0.63

Centre line temperature, T_0 =568.7K

b) Temperature, at a distance of 15mmm from mid plane

x-axis ---- biot number , B_i= hL_c/k =235 *0.03/42.6 =0.165

Curve =X/L_c = $15*10^{-3}/0.03$ =0.5

Fromgraph, $T_i-T_\alpha/T_x-T_\alpha$=0.88

T_x-323 /568.8-323 =0.88 =539.21K

Temperature inside the plate 15mm from mid plane,T_x =539.21K

11. Determine the heat transfer through the composite wall show in the fig-a. take the conductivities of A,B,C,D and E as 50,10,6.67,20,30 w/m k respectively and assume one dimensional heat transfer.

SOLUTION:

R_a=L/KA=0.05/50(1)=$1*10^{-3}(W/K)^{-1}$

R_b=l/KB =0.05/10(.5)=$2*10^{-3}(W/K)^{-1}$

R_C =l/KC =0.05/6.67(.5)=$3*10^{-3}(W/K)^{-1}$

R_d=L/KB =.05/20(.1)=$2.5*10^{-3}(W/K)^{-1}$

R_e=L/KB =.05/30(1)=$1.67*10^{-3}(w/k)^{-1}$

The equivalent resistance for R_b and R_C is

$1/R_F=1/R_B+1/R_C=1/2*10^{-2}+1/3*10^{-2}=0.833/10^{-2}$

$R_F=1.2*10^{-2}(w/k)^{-1}$

$\sum R=R_a+R_f+R_d+R_e=(1+12+2.5+1.67)*10^{-3}=17.17*10^{-3}$

$Q=T_1-T_2/R$ =(800-100)/$17.17*10^{-3}$=$4.07 *10^4$ w =40.7 kw

(1) A steam boiler furnace is made of a layer of fire clay 12.5cm thick and a layer of red bricks 50cm thick .if the wall temperature inside the boiler furnace is 1100^0c and that on outside wall is 50^0c,determine the amount of heat loss per square meter of the furnace wall(k for fire clay=0.533 w/mk and k for red brick=0.7w/mk)

(2) It is a desired to reduce thickness of red brick layer in this furnace to half by filling in the space between the two layer by diatomite whose k=0.113 +0.00023t(w/m k).calculate the thickness of filling to ensure an identical loss of heat for the same outside and inside temperature.

Solution :

(1) R_1 =resistance of fireclay =0.125/0.533 =0.234 (per unit area)

R_2=resistance of fireclay =0.5/0.7 =0.714 (per unit area)

R_1+R_2 =0.234+0.714 =0.948

Heat transfer rate ,q =T_1-T_2/∑R =1100-50/0.948=1107.5 w/m^2

Temperature T_2 can be found as ,q =T_1-T_2/R_1

T_2=T_1-Qr_1

T_2=1100-1107.5(0.234)

=1100-259

T_2=841^{0}c

(2) Since the heat loss of 1107.5 w/mk must remain unchanged ,the temperature at the interface between the two layer of diatomile and red brick is formed as follows.

T_3=T_4+q_1R_2 = 50+(1107.5)(0.25/0.7) =445.5^{0}C

The mean thermal conductivity of diatomile layer is ,

Km =0.113+0.00023(.841+445.5/2)=0.261 w/mk

The thickness of diatomile ,x =T_2-T_3/q Km

=841-445.5/1107.5(0.261)

X=0.0932 m(or)93.2 mm

12.A steel pipe line(K=50w/m k) of I.D 100mm and O.D 110mm is to be covered with two layers of insulation each having a thickness 50mm .the thermal conductivity of the first insulation material is 0.06 w/m k and that of the second is 0.12w/m k .calculate the loss of heat per meter length of pipe and the interface temperature between the two layers of insulation when the temperature of the inside tube surfaces is 250^{0}c and that of the outside surface of the insulation is 50^{0}c.

Solution:

The insulated pipe is shown in fig (a)

T_1 =T_2=250^{0}C;T_3=?

r_1=50mm

,r_2=55mm;K_1=50w/m k,

r_3=105mm;K_2=0.06w/m k,

r_4=115mm;$K_3$0.12w/m k,

Loss of heat per unit length ,(insulation, n=3)

Q/L =2π(T_1-T_4)/ln(r_2/r_1)/K_1+ln(r_3/r_2)/K_2+ln(r_4/r_3)/K_3

=6.28(250-50)/ln(55/50)/50+ln(105/55)/0.06+ln(155/105)/0.12 =89.6 w/m

The interface temperature, T_3 is obtained from the equation

=2π(T_3-T_4)/ ln(r_4/r_3)/K_3

T_3=Q/L. ln(r_4/r_3)/2πK_3 +T_4

=(89.6)ln(155/55)/(0.12)(6.28)+50

T_3=96.3^{0}c

13. Obtain an expression for the general heat conduction equation in cartesian coordinates. [Nov/Dec 2006]

Consider a small rectangular element of sides dx, dy and dz as shown in fig(a)

The energy balance of this rectangular element obtained from first law of thermodynamics

{net heat conducted into element from all The coordinates direction} + {heat generated within the element} = {heat stored in the element} –(1)

Net heat conducted into the element from all the coordinate directions.

Let Q_x be the heat flux in a direction of face ABCD and Q_{x+dx}be the heat flux in the direction of EFGH

The rate of heat flow in to the element in X direction through the face ABCD is

Q_X=$Q_X$dydz=-k_x(∂t/∂x)dy dx

Where, k-thermal conductivity,(w/mk)

T/x –temperature gradient

The rate of heat flow out of the element in x-direction through the face EFGH is ,

Q_X+d_x =Q_X+ ($\partial/\partial x$(Q_X))dx ------(2)

=-$K_x\frac{\partial t}{\partial x}$ dy.dz +$\frac{\partial}{\partial x}$ [-$K_X\frac{\partial T}{\partial x}$dy.dz].dx

=-$K_x\frac{\partial t}{\partial x}$ dy.dz –$\partial/\partial x$[$K_x\frac{\partial t}{\partial x}$]dx.dy.dz------(3)

Sub eqn in 2-3,

Q_x-Q_x+dx =-$K_x\frac{\partial t}{\partial x}$ dy .dz –[-$K_x\frac{\partial t}{\partial x}$ dy.dz – $\partial/\partial x$ [$K_x\frac{\partial t}{\partial x}$]dx.dy.dz]

=- $K_x\frac{\partial t}{\partial x}$ dy.dz+ $K_x\frac{\partial t}{\partial x}$dy.dz+$\partial/\partial x$ [$K_x\frac{\partial t}{\partial x}$]dx.dy

$=\partial/\partial x[K_x\frac{\partial t}{\partial x}]dx.dy.dz$------(4)

Similarly,

$Q_Y-Q_Y+dy=\partial/\partial y[K_y\frac{\partial t}{\partial x}]dx.dy.dz$------(5)

$Q_Z-Q_Z+dz=\partial/\partial z[K_z\frac{\partial t}{\partial x}]\ dx.dy.dz$-------(6)

Adding 4,5,and 6

Net heat conducted $=\partial/\partial x[K_x\frac{\partial t}{\partial x}]dx.dy.dz + \partial/\partial y[K_y\frac{\partial t}{\partial x}]dx.dy.dz + \partial/\partial z[K_z\frac{\partial t}{\partial x}]\ dx.dy.dz$

$=\partial/\partial x[K_x\frac{\partial t}{\partial x}]+\partial/\partial y[K_y\frac{\partial t}{\partial x}]+\partial/\partial z[K_z\frac{\partial t}{\partial x}]\ dx.dy.dz$

Net heat conducted into element from all the coordinate directions.

$=[\partial/\partial x[K_x\partial/\partial y[K_y\frac{\partial t}{\partial x}]+\partial/\partial z[K_z\frac{\partial t}{\partial x}]]dx.dy.dz$-------(7)

Heat stored in the element.

We know that ,

{heat stored in the element } ={mass of the element}*{specific heat of element}*{rise in temperature of element}

$= m*c_p*\frac{\partial T}{\partial t}$

$=\varrho\ dx.dy.dz * c_p*\frac{\partial T}{\partial t}$ [mass =density*volume]

Heat stored in the element $=\varrho\ c_p\frac{\partial T}{\partial t}dx.dy.dz$----- (8)

Heat stored within the element

Heat generated within in the element is given by,

$Q=q\ dx.dy.dz$------- (9)

Sub eqn 7,8,and9 in 1

Eqn (1) $=\partial/\partial x[K_x\frac{\partial t}{\partial x}]dx.dy.dz + \partial/\partial y[K_y\frac{\partial t}{\partial x}]dx.dy.dz + \partial/\partial z[K_z\frac{\partial t}{\partial x}]\ dx.dy.dz+ q\ dx.dy.dz$ $=\varrho c_p\frac{\partial T}{\partial t}dx.dy.dz$

$=\partial/\partial x[K_x\partial/\partial y[K_y\frac{\partial t}{\partial x}]+\partial/\partial z[K_z\frac{\partial t}{\partial x}]+q =\varrho c_p\frac{\partial T}{\partial t}$

Considering the material is isotropic .so,$K_x=K_z=K_y=k$ =constant

$=[\partial^2T/\partial x^2+\partial^2T/\partial y^2+\partial^2T/\partial z^2]K+q = \varrho c_p\frac{\partial T}{\partial t}$

$\partial^2T/\partial x^2+\partial^2T/\partial y^2+\partial^2T/\partial z^2+q/K = \varrho c_p\frac{}{K}\frac{\partial T}{\partial t}$

$\partial^2T/\partial x^2+\partial^2T/\partial y^2+\partial^2T/\partial z^2+q/K=1/\alpha.\frac{\partial T}{\partial t}$--------- (10)

It is a general three dimensional heat conduction eqn in Cartesian coordinates.

Where,α=thermal diffusivity =K/pc_p m^3/s

Thermal diffusivity is nothing but how fast heat is diffused through a material during of temperature with time.

Note :

Case 1: no heat sources.

In the absences of internal heat generation ,eqn (10)reduces to

$\partial^2T/\partial x^2+\partial^2T/\partial y^2+\partial^2T/\partial z^2=1/\alpha.\frac{\partial T}{\partial t}$-------- (11)

This equation is known as diffusion eqn (or)fouriereqn

Case2: steady state conditions

In steady state condition, the temperature does not change with time .so $\frac{\partial T}{\partial t}$ =0. The eqn conduction eqn (10) reduces to

$\partial^2T/\partial x^2+\partial^2T/\partial y^2+\partial^2T/\partial z^2+q/K$ =0----------- (12)

This known as poissonseqn

In absence of internal heat generation, eqn (12) becomes $\partial^2T/\partial x^2+\partial^2T/\partial y^2+\partial^2T/\partial z^2=0$ or $\nabla^2T=0$

This eqn is known as laplaceeqn

Case 3: one dimensional steady state heat condition

If the temperature varies only in x-direction, the eqn (10) reduces to

$\partial^2T/\partial x^2+ q/K=0$ --------- (14)

In absence of internal heat generation, eqn(14) becomes

$\partial^2T/\partial x^2=0$ ------ (15)

Case4 : Two dimensional steady state heat condition

If the temperature varies only in the x and y directions, the eqn (10) becomes $\partial^2T/\partial x^2+\partial^2T/\partial y^2+q/K=0$ --------- (16)

In the absence of internal heat generation,eqn(16) reduces to

$\partial^2T/\partial x^2+\partial^2T/\partial y^2=0$----- (17)

Case5: unsteady state, one dimensional, without internal heat generation

In unsteady state, the temperature changes with time ,i.e$\partial T/\partial t \neq 0$. So,the general conduction eqn (10) reduces to $\partial^2T/\partial x^2=1/\alpha.\partial T/\partial t$-----(18)

14. a) An exterior wall of a house is covered by 10mm common bricks (K=0.7w/m k) followed by a 4cm layer of gypsum plaster (K=0.48w/m k) .what thickness of loosely packed insulation (K=0.065w/m k)should be added to reduce the heat loss through the wall by 80%? [May-2004]

Given data:

Thickness of brick, L1=10cm =0.1m

Thermal conductivity of brick, K1=0.7w/m k

Thickness of gypsum, L2=4CM =0.04m

Thermal conductivity of gypsum, K2=0.48w/m k

Thermal conductivity of insulation, K3=0.065w/m k

To find:

Thickness of insulation to reduce the heat loss through the wall by80% (L_3)

SOLUTION:

Heat flow rate, $Q=\Delta T_{overall\ /}R$ [from HMT data book]

Where,

$R = 1/A\ [1/h_a+l_1/k_1+l_2/k_2+l_3/k_3+1/h_p]$

[The time h_a and h_b are not given .so neglect are term

$R = 1/A\ [1/h_a+l_1/k_1+l_2/k_2+l_3/k_3]$

Considering two slabs (i.e) neglect l_3 term [$A=1m^2$]

$Q=\Delta T/\ l_1/k_1+l_2/k_2$

$1000= \Delta T/0.1/0.7+0.04/0.48$ [assume heat transfer (Q)=100W]

$\Delta T= 22.619$ K

Heat loss is reduced by 80% due to insulation .so heat transfer is 20W

$20=\Delta T/1/A\ [1/h_a+l_1/k_1+l_2/k_2+l_3/k_3]$

$20 =22.619/1/1\ [0.1/0.7+0.04/0.48+l_3/0.065]$

$L_3=0.0588$ m

Result :

Thickness of insulation, l3**=0.0588m**

15. A plane wall 10cm thick generator heat at rate of $4*10^4$wm^3 when a electric current is passed through it. the convective heat transfer coefficient between each face of the wall and ambient air is50 w/m^3.determine (a) surface temperature (b) the maximum air temperature the wall assume that ambient air temperature to be 20^{0}c and the thermal conductivity of the wall material to be 15 w/m k [April- 98]

Given data:

Thickness, l=10 cm =0.10 m

Heat generation, q=$4*10^4$w/m^3

Convective heat transfer coefficient ,h =50w/m k

Ambient air temperature, T_∞= 20^{0}c +273 =293 K,

Thermal conductivity, K =15w/m k

To find:

1) Surface temperature (2) maximum temperature in the wall

Solution:

Surface wall temperature, $T_w = T_\alpha + (Q^O L/2H)$

$=293+(4*10^4*0.10)/(2*50)$

$T_W=60^0C=33K$

MAXIMUM TEMPARATURE,$T_{MAX}=T_W+(Q^0L^2/8K)$

$=333+(4*10^4*1.0^2)/(8*15)$

T_{MAX}=336.3K(OR)63.3^{0}C

RESULT

Surface temperature, T_W=333K

Maximum temperature, T_{MAX}=336.3K

16. A cylinder 1m long and 5cm in diameter is placed in an atmosphere at 45^{0}c.it is provided with 10 longitudinal straight fins of material having k=120 w/mk. the height of 0.76 mm thick fins is 1.27 cm from the cylinder surface. the heat transfer coefficient between cylinder and atmosphere air is 17 k/m^{2}k.calculate the rate of heat transfer and the temperature at the end of fins it surface temperature cylinder is 150^{0}c.

Given data:

Length of the engine cylinder, l_{cy}=1m

Diameter of the cylinder ,d=5cm=0.05m

Atmosphere temperature ,$T_\alpha=45^0c+273=318k$

Number of fins=10

Thermal conductivity of fins, k=120k/m k

Thickness of the fin, $t=0.76mm=0.76*10^{-3}m$

Length(height) of the fin, $l_f=1.27cm =1.27*10^{-2}m$

Heat transfer co efficient ,$h=17\ w/m^2k$

Cylindrical surface temperature (or)base temperature ,$t_b=150^0c+273=423k$

To find:

1) Rate of heat transfer, q
2) Temperature at the end of the fin

Solution:

Length of the fin is 1.27cm .so ,this is short fin ,assuming that the fin end is insulated .

We know that,

Heat transfer,$Q =(hpka)^{1/2}(t_b-t\infty)\tan h(mL_f)$-----(1)

Where,

Perimeter, p =2*length of the cylinder =2*1 =2 m

Area, A =length of the cylinder *thickness $=1*0.76*10^{-3}$

$A=0.76*10^{-3}m^2$

$m =\sqrt{Hp/Ka} =\sqrt{17*2/120*0.76*10^{-3}}$

$=19.30m^{-1}$

Eqn (1)= $(hpka)^{1/2}(t_b-t\infty)\tan h(mL_f)$

$=[17*2*120*10^{-3}]^{1/2}(423-318)*\tan(19.32*1.27*10^{-2})$

$Q_1=44.3W$

Heat transfer per fin =44.3W

For 10 fins, heat transfer =44.3*10 =443W

$Q_1=44.3W$-----(2)

Heat transfer from unfinned surface due to convection is $Q_2=hA\Delta T$

$=h\ [\pi dL_{CY}-10*t*L_f](T_b-T\infty)$

[Area of unfinned surface =area of cylinder – area of fin

$=17*[(\pi*0.051)-(w*0.76*10^{-3}*1.27*10^{-2})]*(423-318)$

Q_2=280.21w

So, total heat transfer ,$Q=Q_1+Q_2$

=443+280.21=723.21W

We know that,

Temperature distribution [short fin, end insulated]

$T-T\infty/T_b-T\infty=\cosh [m(L_f-x)/\cos h(mL_f)$

We need temperature at the end of fin ,so put x=L

$=\cosh[m(L-L)]/\cosh (19.30*1.27*10^{-2})$

T-318/423-318 =1/1.030 =419.94 k

Result :

Heat transfer, Q =723.21w

Temperature at the end of the fin ,T=419.94K

17.A turbine blade 8cm long made of stainless steel (K=32w/mk) has cross sectional area of 4.75cm^2 and a perimeter of 12cm .the base temperature of the blade is 600^{0}c .find the quantity of heat given to blade if in the blade is exposed to hot gases850^{0}c .take heat transfer coefficient to be 465 w/m^{2}k

Given data :

Length of the blade ,L=8cm =0.08m

Thermal conductivity ,K= 32w /mk

Area ,A =4.75cm^2=4.75*10^{-4}m^2

Perimeter ,P=12cm=0.12m

Base temperature ,T_b= 600^{0}C +273=873k

Hot gas temperature ,$T\infty$=850^{0}c+273=1123k

Heat transfer coefficient ,h=465w/m^{2}k

To find :

Since the blade length is 8cm,it is treated as short fin .

Assume end is insulated .

Heat transfer [short fin ,end insulated]

$Q=(HPKA)^{1/2}(T_b-T\infty)\tanh(mL)$

Where,

$m=\sqrt{Hp/Ka} =\sqrt{465*0.12/32*4.75*10^{-4}}=60.5m^{-1}$

eqn 1=q=$(465*0.12*32*4.75*10^{-4})^{1/2}$*(873-112.3)*tan h(60.5*0.08)

q=-230.2w

[-ve sign indicates that heat flows from gas to turbine blades]

18. Slab of aluminum 10cm thick is originally at a temperature of 500^0c.it is suddenly immersed in a liquid at 100^0c resulting in a heat transfer coefficient of 1200w/m^2k.determine the temperature of the center line and the surface I min after the immersion. also calculate the total thermal energy removed per unit area of the slab during this period. the properties for the aluminum for the given conditions are A=8.4*$10^{-5}$$m^2$/s, k=215w/mk ,p=2700kg/$m^3$,c=0.9kj/kg k

Given data:

Thickness, l=10cm=0.1m

Initial temperature, ti=500^0c+273k=773k

Final temperature, tα=100^0c+273=373k

Properties of aluminumare ,

Density, ϱ=2700 kg/m^2

Thermal diffusivity=8.4*10^{-}5m^2/s

Thermal conductivity=215w/mk

Specific heat,c_p=0.9kj/kg k

To find

1) Temperature at the center line after 1min

2) Temperature at the surface

3) Total thermal energy removed per unit area

Solution:

We know that,

Characteristics length of slab ,L_c=L/2=0.1/2=0.05m

Biot number, Bi =hL_c/k=1200*0.05/215=0.279

Biot number value is in between 0.1 and 100 (i.e)0.1<Bi<100 .so ,this is infinite solid type problem

Case(1):

To calculate mid plane temperature for infinite plate ,referHMTdata book –heister chart

x-axisfourier number =$\alpha t/L_c^2$=8.4*10^{-5}*60/$(0.05)^2$=2.016

curves value hL_c/K =1200*0.05/215 =0.219

x axis value is 2.016 curve value is 0.279 .from that we can find corresponding y axis value is 0.64

y axis $=T_0-T\infty / \ T_i-T\infty$ =0.64

T_0-373/773-373 =0.64

T_0=629 K

Centre line temperature, T_0=629K

Case(2)

CURVE x/L_c= 0.05/0.05 =1

x-axis value is 0.279 curve value is 1 .from that we can find corresponding y –axis value is 0.88

y-axis $=T_x-T\infty / \ T_0-T\infty$ =0.88

T_x-273/629-373=0.88

T_x=598.28 K

Temperature at a surface ,T_x=598.28K

Case(3)

Total thermal energy removed or total heat energy removed

x-axisfourier number $=h^2\alpha t/k^2=(1200)^2*8.4*10^{-5}*60/(215)^2=0.517$

curve value $=hL_c/K$= 1200*0.05/215= 0.279

x-axis value is 0.517 ,curve value is0.279 .from that we can

find corresponding y-axis value is 0.34

we know that

$Q_0=\varrho C_pL[T_i-T\infty]$

$=2700*0.9*10^3 0.10*[773-373]=97.2*10^6 j/m^2$

From graph ,we know that

Q/Q_0=0.34

$Q=0.34*97.2*10^6=33.04*10^6 j/m^2$

Total Thermal energy removed per unit area Q =$33.04*10^6 j/m^2$

19. A wall is constructed of several layers. The first layer consists of masonry brick 20 cm. thick of thermal conductivity 0.66 W/mK, the second layer consists of 3 cm thick mortar of thermal conductivity 0.6 W/mK, the third layer consists of 8 cm thick lime stone of

thermal conductivity 0.58 W/mK and the outer layer consists of 1.2 cm thick plaster of thermal conductivity 0.6 W/mK. The heat transfer coefficient on the interior and exterior of the wall are 5.6 W/m^2K and 11 W/m^2K respectively. Interior room temperature is 22°C and outside air temperature is -5°C.

Calculate

- **a) Overall heat transfer coefficient**
- **b) Overall thermal resistance**
- **c) The rate of heat transfer**
- **d) The temperature at the junction between the mortar and the limestone.**

Given Data

Thickness of masonry L_1 = 20cm = 0.20 m

Thermal conductivity K_1 = 0.66 W/mK

Thickness of mortar L_2 = 3cm = 0.03 m

Thermal conductivity of mortar K_2 = 0.6 W/mK

Thickness of limestone L_3 = 8 cm = 0.08 m

Thermal conductivity K_3 = 0.58 W/mK

Thickness of Plaster L_4 = 1.2 cm = 0.012 m

Thermal conductivity K_4 = 0.6 W/mK

Interior heat transfer coefficient h_a = 5.6 W/m^2K

Exterior heat transfer co-efficient h_b = 11 W/m^2K

Interior room temperature T_a = 22°C + 273 = 295 K

Outside air temperature T_b = -5°C + 273 = 268 K.

Solution:

Heat flow through composite wall is given by

$Q = \frac{\Delta T_{overall}}{R}$ [From equation (13)] (or) [HMT Data book page No. 34]

Where, $\Delta T = T_a - T_b$

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}}$$

$$\Rightarrow Q / A = \frac{295 - 268}{\frac{1}{5.6} + \frac{0.20}{0.66} + \frac{0.03}{0.6} + \frac{0.08}{0.58} + \frac{0.012}{0.6} + \frac{1}{11}}$$

$$\boxed{\text{Heat transfer per unit area } Q/A = 34.56 \text{ W/m}^2}$$

We know, Heat transfer Q = UA (T_a – T_b) [From equation (14)]

Where U – overall heat transfer co-efficient

$$\Rightarrow U = \frac{Q}{A \times (T_a - T_b)}$$

$$\Rightarrow U = \frac{34.56}{295 - 268}$$

$$\boxed{\text{Overall heat transfer co-efficient } U = 1.28 \text{ W/m}^2 K}$$

We know

Overall Thermal resistance (R)

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{L_4}{K_4 A} + \frac{1}{h_b A}$$

For unit Area

$$R = \frac{1}{h_a} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{L_4}{K_4} + \frac{1}{h_b}$$

$$= \frac{1}{56} + \frac{0.20}{0.66} + \frac{0.03}{0.6} + \frac{0.08}{0.58} + \frac{0.012}{0.6} + \frac{1}{11}$$

$$\boxed{R = 0.78 \ K/W}$$

Interface temperature between mortar and the limestone T_3

Interface temperatures relation

$$\Rightarrow Q = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_4}{R_3} = \frac{T_4 - T_5}{R_4} = \frac{T_5 - T_b}{R_b}$$

$$\Rightarrow Q = \frac{T_a - T_1}{R_a}$$

$$Q = \frac{295 - T_1}{1/h_a A} \qquad \left[\because R_a = \frac{1}{h_a A}\right]$$

$$\Rightarrow Q/A = \frac{295 - T_1}{1/h_a}$$

$$\Rightarrow 34.56 = \frac{295 - T_1}{1/5.6}$$

$$\Rightarrow \boxed{T_1 = 288.8\ K}$$

$$\Rightarrow Q = \frac{T_1 - T_2}{R_1}$$

$$Q = \frac{288.8 - T_2}{\frac{L_1}{K_1 A}} \qquad \left[\because R_1 = \frac{L_1}{k_1 A}\right]$$

$$\Rightarrow Q/A = \frac{288.8 - T_2}{\frac{L_1}{K_1}}$$

$$\Rightarrow 34.56 = \frac{288.8 - T_2}{\frac{0.20}{0.66}}$$

$$\Rightarrow \boxed{T_2 = 278.3\ \text{K}}$$

$$\Rightarrow Q = \frac{T_2 - T_3}{R_2}$$

$$Q = \frac{278.3 - T_3}{\frac{L_2}{K_2 A}} \qquad \left[\because R_2 = \frac{L_2}{K_2 A}\right]$$

$$\Rightarrow Q/A = \frac{278.3 - T_3}{\frac{L_2}{K_2}}$$

$$\Rightarrow 34.56 = \frac{278.3 - T_3}{\frac{0.03}{0.6}}$$

$$\Rightarrow \boxed{T_3 = 276.5\ \text{K}}$$

Temperature between Mortar and limestone (T_3 is 276.5 K)

20. A furnace wall made up of 7.5 cm of fire plate and 0.65 cm of mild steel plate. Inside surface exposed to hot gas at 650°C and outside air temperature 27°C. The convective heat transfer co-efficient for inner side is 60 W/m^{2}K.The convective heat transfer co-efficient for outer side is 8W/m^{2}K. Calculate the heat lost per square meter area of the furnace wall and also find outside surface temperature.

Given Data

Thickness of fire plate L_1 = 7.5 cm = 0.075 m

Thickness of mild steel L_2 = 0.65 cm = 0.0065 m

Inside hot gas temperature T_a = 650°C + 273 = 923 K

Outside air temperature T_b = 27°C + 273 = 300°K

Convective heat transfer co-efficient for

Inner side $h_a = 60 W/m^2K$

Convective heat transfer co-efficient for

Outer side $h_b = 8\ W/m^2K$.

Solution:

(i) Heat lost per square meter area (Q/A)

Thermal conductivity for fire plate

$K_1 = 1035 \times 10^{-3}$ W/mK [From HMT data book page No.11]

Thermal conductivity for mild steel plate

K_2 = 53.6W/mK [From HMT data book page No.1]

Heat flow $Q = \frac{\Delta T_{overall}}{R}$, Where $\Delta T = T_a - T_b$

$$R = \frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}}$$ [The term L_3 is not given so neglect that term]

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_b A}}$$

The term L_3 is not given so neglect that term]

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{h_a A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{1}{h_b A}}$$

$$Q / A = \frac{923 - 300}{\frac{1}{60} + \frac{0.075}{1.035} + \frac{0.0065}{53.6} + \frac{1}{8}}$$

$$\boxed{Q / A = 2907.79\ W / m^2}$$

(ii) **Outside surface temperature T_3**

We know that, Interface temperatures relation

$$Q = \frac{T_a - T_b}{R} = \frac{T_a - T_1}{R_a} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_2} = \frac{T_3 - T_b}{R_b} ......(A)$$

$$(A) \Rightarrow Q = \frac{T_3 - T_b}{R_b}$$

where

$$R_b = \frac{1}{h_b A}$$

$$\Rightarrow Q = \frac{T_3 - T_b}{\frac{1}{h_b A}}$$

$$\Rightarrow Q/A = \frac{T_3 - T_b}{\frac{1}{h_b}}$$

$$\Rightarrow 2907.79 = \frac{T_3 - 300}{\frac{1}{8}}$$

$$\boxed{T_3 = 663.473\ K}$$

21. A steel tube (K = 43.26 W/mK) of 5.08 cm inner diameter and 7.62 cm outer diameter is covered with 2.5 cm layer of insulation (K = 0.208 W/mK) the inside surface of the tube receivers heat from a hot gas at the temperature of 316°C with heat transfer co-efficient of 28 W/m^{2}K. While the outer surface exposed to the ambient air at 30°C with heat transfer co-efficient of 17 W/m^{2}K. Calculate heat loss for 3 m length of the tube. [May-June-2009]

Given

Steel tube thermal conductivity K_1 = 43.26 W/mK
Inner diameter of steel d_1 = 5.08 cm = 0.0508 m
Inner radius r_1 = 0.0254 m
Outer diameter of steel d_2 = 7.62 cm = 0.0762 m
Outer radius r_2 = 0.0381 m
Radius r_3 = r_2 + thickness of insulation
Radius r_3 = 0.0381 + 0.025 m r_3 = 0.0631 m
Thermal conductivity of insulation K_2 = 0.208 W/mK
Hot gas temperature T_a = 316°C + 273 = 589 K
Ambient air temperature T_b = 30°C + 273 = 303 K
Heat transfer co-efficient at inner side h_a = 28 W/m^{2}K
Heat transfer co-efficient at outer side h_b = 17 W/m^{2}K
Length L = 3 m

Solution :

Heat flow $Q = \frac{\Delta T_{overall}}{R}$ [From equation No.(19) or HMT data book Page No.35]

Where Δ T = $T_a - T_b$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{K_1} In\left[\frac{r_2}{r_1}\right] + \frac{1}{K_2} In\left[\frac{r_3}{r_2}\right] + \frac{1}{K_3} In\left[\frac{r_4}{r_3}\right] + \frac{1}{h_b r_4}\right]$$

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{K_1} In\left[\frac{r_2}{r_1}\right] + \frac{1}{K_2} In\left[\frac{r_3}{r_2}\right] + \frac{1}{K_3} In\left[\frac{r_4}{r_3}\right] + \frac{1}{h_b r_4}\right]}$$

[The terms K_3 and r_4 are not given, so neglect that terms]

$$\Rightarrow Q = \frac{T_a - T_b}{\frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{K_1} In\left[\frac{r_2}{r_1}\right] + \frac{1}{K_2} In\left[\frac{r_3}{r_2}\right] + \frac{1}{h_b r_3}\right]}$$

$$\Rightarrow Q = \frac{589 - 303}{\frac{1}{2\pi \times 3}\left[\frac{1}{28 \times 0.0254} + \frac{1}{43.26} In\left[\frac{0.0381}{0.0254}\right] + \frac{1}{0.208} In\left[\frac{0.0631}{0.0381}\right] + \frac{1}{17 \times 0.0631}\right]}$$

$$\boxed{Q = 1129.42 \text{ W}}$$

Heat loss Q = 1129.42 W.

22. **Derive an expression of Critical Radius of Insulation For A Cylinder.**

Consider a cylinder having thermal conductivity K. Let r_1 and r_0 inner and outer radii of insulation.

Heat transfer $Q = \frac{T_i - T_\infty}{\frac{In\left[\frac{r_0}{r_1}\right]}{2\pi K L}}$ [From equation No.(3)]

Considering h be the outside heat transfer co-efficient.

$$\therefore Q = \frac{T_i - T_\infty}{\frac{In\left[\frac{r_0}{r_1}\right]}{2\pi K L} + \frac{1}{A_0 h}}$$

Here $A_0 = 2\pi r_0 L$

$$\Rightarrow Q = \frac{T_i - T_\infty}{\frac{In\left[\frac{r_0}{r_1}\right]}{2\pi K L} + \frac{1}{2\pi r_0 L h}}$$

To find the critical radius of insulation, differentiate Q with respect to r_0 and equate it to zero.

$$\Rightarrow \frac{dQ}{dr_0} = \frac{0-(T_i - T_\infty)\left[\frac{1}{2\pi K L r_0} - \frac{1}{2\pi h L r_0^2}\right]}{\frac{1}{2\pi K L}\ln\left[\frac{r_0}{r_1}\right] + \frac{1}{2\pi h L r_0}}$$

$$\text{since } (T_i - T_\infty) \neq 0$$

$$\Rightarrow \frac{1}{2\pi K L r_0} - \frac{1}{2\pi h L r_0^2} = 0$$

$$\Rightarrow \boxed{r_0 = \frac{K}{h} = r_c}$$

23. A wire of 6 mm diameter with 2 mm thick insulation (K = 0.11 W/mK). If the convective heat transfer co-efficient between the insulating surface and air is 25 W/m²L, find the critical thickness of insulation. And also find the percentage of change in the heat transfer rate if the critical radius is used.

Given Data

d_1= 6 mm
r_1 = 3 mm = 0.003 m
$r_2 = r_1 + 2 = 3 + 2 = 5$ mm = 0.005 m
K = 0.11 W/mK
h_b = 25 W/m^{2}K

Solution :

1. Critical radius $r_c = \frac{K}{h}$ [From equation No.(21)]

$$r_c = \frac{0.11}{25} = 4.4 \times 10^{-3} \text{m}$$

$$\boxed{r_c = 4.4 \times 10^{-3} \text{m}}$$

Critical thickness = $r_c - r_1$

$$= 4.4 \times 10^{-3} - 0.003$$

$$= 1.4 \times 10^{-3} \text{ m}$$

$$\boxed{\text{Critical thickness } t_c = 1.4 \times 10^{-3} \text{ (or) } 1.4 \text{ mm}}$$

2. Heat transfer through an insulated wire is given by

$$Q_1 = \frac{T_a - T_b}{\frac{1}{2\pi L}\left[\frac{\ln\left[\frac{r_2}{r_1}\right]}{K_1} + \frac{1}{h_b r_2}\right]}$$

[From HMT data book Page No.35]

$$= \frac{2\pi L\ (T_a - T_b)}{\left[\frac{\ln\left[\frac{0.005}{0.003}\right]}{0.11} + \frac{1}{25 \times 0.005}\right]}$$

$$Q1 = \frac{2\pi L\ (T_a - T_b)}{12.64}$$

Heat flow through an insulated wire when critical radius is used is given by

$$Q_2 = \frac{T_a - T_b}{\frac{1}{2\pi L}\left[\frac{\ln\left[\frac{r_c}{r_1}\right]}{K_1} + \frac{1}{h_b r_c}\right]} \qquad [r_2 \rightarrow r_c]$$

$$= \frac{2\pi L\ (T_a - T_b)}{\frac{\ln\left[\frac{4.4 \times 10^{-3}}{0.003}\right]}{0.11} + \frac{1}{25 \times 4.4 \times 10^{-3}}}$$

$$Q_2 = \frac{2\pi L\ (T_a - T_b)}{12.572}$$

$\therefore$ Percentage of increase in heat flow by using

Critical radius = $\frac{Q_2 - Q_1}{Q_1} \times 100$

$$= \frac{\frac{1}{12.57} - \frac{1}{12.64} \times 100}{\frac{1}{12.64}}$$

$= 0.55\%$

24. Analuminum alloy fin of 7 mm thick and 50 mm long protrudes from a wall, which is

maintained at 120°C. The ambient air temperature is 22°C. The heat transfer coefficient and conductivity of the fin material are 140 W/m^{2}K and 55 W/mK respectively. Determine

1. Temperature at the end of the fin.
2. Temperature at the middle of the fin.
3. Total heat dissipated by the fin.

Given

Thickness t = 7mm = 0.007 m
Length L= 50 mm = 0.050 m
Base temperature T_b = 120°C + 273 = 393 K
Ambient temperature T_∞ = 22° + 273 = 295 K
Heat transfer co-efficient h = 140 W/m^{2}K
Thermal conductivity K = 55 W/mK.

Solution :

Length of the fin is 50 mm. So, this is short fin type problem. Assume end is insulated.

We know

Temperature distribution [Short fin, end insulated]

$$\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cos h\ m\ [L - x]}{\cos h\ (mL)} \quad ....(A)$$

[From HMT data book Page No.41]

(i) Temperature at the end of the fin, Put x = L

$$(A) \Rightarrow \quad \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cos h\ m\ [L - L]}{\cos h\ (mL)}$$

$$\Rightarrow \quad \frac{T - T_\infty}{T_b - T_\infty} = \frac{1}{\cos h\ (mL)} \quad ...(1)$$

where

$$m = \sqrt{\frac{hP}{KA}}$$

P = Perimeter = $2 \times L$ (Approx)

$= 2 \times 0.050$

$\boxed{P = 0.1\ m}$

A – Area = Length × thickness = 0.050 × 0.007

$$\boxed{A = 3.5 \times 10^{-4} m^2}$$

$$\Rightarrow \quad m = \sqrt{\frac{hP}{KA}}$$

$$= \sqrt{\frac{140 \times 0.1}{55 \times 3.5 \times 10^{-4}}}$$

$$\boxed{m = 26.96}$$

$$(1) \quad \Rightarrow \quad \frac{T - T_\infty}{T_b - T_\infty} = \frac{1}{\cos h\,(26.9 \times 0.050)}$$

$$\Rightarrow \quad \frac{T - T_\infty}{T_b - T_\infty} = \frac{1}{2.05}$$

$$\Rightarrow \quad \frac{T - 295}{393 - 295} = \frac{1}{2.05}$$

$$\Rightarrow \quad T - 295 = 47.8$$

$$\Rightarrow \quad \boxed{T = 342.8 \text{ K}}$$

$$\boxed{\text{Temperature at the end of the fin } T_{x=L} = 342.8 \text{ K}}$$

(ii) Temperature of the middle of the fin,

Put x = L/2 in Equation (A)

$$(A) \quad \Rightarrow \quad \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cos hm\,[L - L/2]}{\cos h\,(mL)}$$

$$\Rightarrow \quad \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cos h\, 26.9\left[0.050 - \frac{0.050}{2}\right]}{\cos h\,[26.9 \times (0.050)]}$$

$$\Rightarrow \quad \frac{T - 295}{393 - 295} = \frac{1.234}{2.049}$$

$$\Rightarrow \quad \frac{T - 295}{393 - 295} = 0.6025$$

$$\boxed{T = 354.04 \text{ K}}$$

Temperature at the middle of the fin

$$\boxed{T_{x=L/2} = 354.04 \text{ K}}$$

(iii) Total heat dissipated

[From HMT data book Page No.41]

$\Rightarrow \quad Q = (hPKA)^{1/2}(T_b - T_\infty)\tan h\,(mL)$

$\Rightarrow \quad [140 \times 0.1 \times 55 \times 3.5 \times 10^{-4}]^{1/2} \times (393 - 295)$

$\quad \times \tan h\,(26.9 \times 0.050)$

$\boxed{Q = 44.4\ W}$

25.A copper plate 2 mm thick is heated up to 400°C and quenched into water at 30°C. Find the time required for the plate to reach the temperature of 50°C. Heat transfer co-efficient is 100 W/m^{2}K. Density of copper is 8800 kg/m^3. Specific heat of copper = 0.36 kJ/kg K.
Plate dimensions = 30 × 30 cm.

Given

Thickness of plate L = 2 mm = 0.002 m
Initial temperature T_0 = 400°C + 273 = 673 K
Final temperature T = 30°C + 273 = 303 K
Intermediate temperature T = 50°C + 273 = 323 K
Heat transfer co-efficient h = 100 W/m^{2}K
Density ρ = 8800 kg/m^3
Specific heat C_ρ= 360 J/kg k
Plate dimensions = 30 × 30 cm

To find

Time required for the plate to reach 50°C.
[From HMT data book Page No.2]

Solution:

Thermal conductivity of the copper K = 386 W/mK
For slab,

Characteristic length $L_c = \frac{L}{2}$

$= \frac{0.002}{2}$

$\boxed{L_c = 0.001\ m}$

We know,

Biot number $B_i = \frac{hL_c}{K}$

$= \frac{100 \times 0.001}{386}$

$B_i = 2.59 \times 10^{-4} < 0.1$

Biot number value is less than 0.1. So this is lumped heat analysis type problem.

For lumped parameter system,

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-hA}{C_\rho \times V \times \rho} \times t\right]} \quad ....(1)$$

[From HMT data book Page No.48]

We know,

Characteristics length $L_c = \frac{V}{A}$

$$(1) \Rightarrow \quad \frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-h}{C_\rho \times L_c \times \rho} \times t\right]}$$

$$\Rightarrow \quad \frac{323 - 303}{673 - 303} = e^{\left[\frac{-100}{360 \times 0.001 \times 8800} \times t\right]}$$

$$\Rightarrow \quad \boxed{t = 92.43 \text{ s}}$$

Time required for the plate to reach 50°C is 92.43 s.

26. A steel ball (specific heat = 0.46 kJ/kgK. and thermal conductivity = 35 W/mK) having 5 cm diameter and initially at a uniform temperature of 450°C is suddenly placed in a control environment in which the temperature is maintained at 100°C. Calculate the time required for the balls to attained a temperature of 150°C. Take h = 10W/m^{2}K.

Given

Specific heat C_ρ = 0.46 kJ/kg K = 460 J/kg K
Thermal conductivity K = 35 W/mK
Diameter of the sphere D = 5 cm = 0.05 m
Radius of the sphere R = 0.025 m
Initial temperature T_0 = 450°C + 273 = 723 K
Final temperature T_∞ = 100°C + 273 = 373 K
Intermediate temperature T = 150°C + 273 = 423 K
Heat transfer co-efficient h = 10 W/m^{2}K

To find

Time required for the ball to reach 150°C

[From HMT data book Page No.1]

Solution

Density of steel is 7833 kg/m^3

$$\boxed{\rho = 7833 \text{ kg/m}^3}$$

For sphere,

Characteristic Length $L_c = \frac{R}{3}$

$$= \frac{0.025}{3}$$

$$\boxed{L_c = 8.33 \times 10^{-3} \text{ m}}$$

We know,

Biot number $B_i = \frac{hL_c}{K}$

$$= \frac{10 \times 8.3 \times 10^{-3}}{35}$$

$B_i = 2.38 \times 10^{-3} < 0.1$

Biot number value is less than 0.1. So this is lumped heat analysis type problem.

For lumped parameter system,

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-hA}{C_\rho \times V \times \rho} \times t\right]} \quad \text{..........(1)}$$

[From HMT data book Page No.48]

We know,

Characteristics length $L_c = \frac{V}{A}$

$$(1) \Rightarrow \frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-h}{C_\rho \times L_c \times \rho} \times t\right]}$$

$$\Rightarrow \frac{423 - 373}{723 - 373} = e^{\left[\frac{-10}{460 \times 8.33 \times 10^{-3} \times 7833} \times t\right]}$$

$$\Rightarrow \ln \frac{423 - 373}{723 - 373} = \frac{-10}{460 \times 8.33 \times 10^{-3} \times 7833} \times t$$

$$\Rightarrow \boxed{t = 5840.54 \text{ s}}$$

Time required for the ball to reach 150°C is 5840.54 s.

27.. Alloy steel ball of 2 mm diameter heated to 800°C is quenched in a bath at 100°C. The material properties of the ball are K = 205 kJ/m hr K, ρ = 7860 kg/m^3, C_ρ = 0.45 kJ/kg K, h = 150 KJ/ hr m^2 K. Determine (i) Temperature of ball after 10 second and (ii) Time for ball to cool to 400°C.

Given

Diameter of the ball D = 12 mm = 0.012 m

Radius of the ball R = 0.006m

Initial temperature T_0 = 800°C + 273 = 1073 K

Final temperature T_∞ = 100°C + 273 = 373 K

Thermal conductivity K = 205 kJ/m hr K

$= \frac{205 \times 1000 J}{3600\ s\ mK}$

$= 56.94\ W/mK$ $[\because\ J/s = W]$

Density ρ = 7860 kg/m^3

Specific heat C_ρ = 0.45 kJ/kg K

= 450 J/kg K

Heat transfer co-efficient h = 150 kJ/hr m^2 K

$= \frac{150 \times 1000 J}{3600\ s\ m^2 K}$

$= 41.66\ W/m^2 K$

Solution

Case (i) Temperature of ball after 10 sec.

For sphere,

Characteristic Length $L_c = \frac{R}{3}$

$= \frac{0.006}{3}$

$\boxed{L_c = 0.002\ m}$

We know,

Biot number $B_i = \frac{hL_c}{K}$

$= \frac{41.667 \times 0.002}{56.94}$

$B_i = 1.46 \times 10^{-3} < 0.1$

Biot number value is less than 0.1. So this is lumped heat analysis type problem.

For lumped parameter system,

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-hA}{C_\rho \times V \times \rho} \times t\right]} \quad ....(1)$$

[From HMT data book Page No.48]

We know,

Characteristics length $L_c = \frac{V}{A}$

$$(1) \Rightarrow \quad \frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-h}{C_\rho \times L_c \times \rho} \times t\right]} \quad ....(2)$$

$$\Rightarrow \quad \frac{T - 373}{1073 - 373} = e^{\left[\frac{-41.667}{450 \times 0.002 \times 7860} \times 10\right]}$$

$$\Rightarrow \quad \boxed{T = 1032.95\ K}$$

Case (ii) Time for ball to cool to 400°C

$\therefore T = 400°C + 273 = 673\ K$

$$(2) \Rightarrow \quad \frac{T - T_\infty}{T_0 - T_\infty} = e^{\left[\frac{-h}{C_\rho \times L_c \times \rho} \times t\right]} \quad ....(2)$$

$$\Rightarrow \quad \frac{673 - 373}{1073 - 373} = e^{\left[\frac{-41.667}{450 \times 0.002 \times 7860} \times t\right]}$$

$$\Rightarrow \quad \ln\left[\frac{673 - 373}{1073 - 373}\right] = \frac{-41.667}{450 \times 0.002 \times 7860} \times t$$

$$\Rightarrow \quad \boxed{t = 143.849\ s}$$

28. A large steel plate 5 cm thick is initially at a uniform temperature of 400°C. It is suddenly exposed on both sides to a surrounding at 60°C with convective heat transfer co-efficient of 285 W/m^{2}K. Calculate the centre line temperature and the temperature inside the plate 1.25 cm from themed plane after 3 minutes.

Take K for steel = 42.5 W/mK, α for steel = 0.043 m^2/hr.

Given

Thickness L = 5 cm = 0.05 m

Initial temperature T_i = 400°C + 273 = 673 K

Final temperature T_∞ = 60°C + 273 = 333 K

Distance x = 1.25 mm = 0.0125 m

Time t = 3 minutes = 180 s

Heat transfer co-efficient h = 285 W/m^{2}K

Thermal diffusivity α = 0.043 m^2/hr = 1.19×10^{-5} m^2/s.

Thermal conductivity K = 42.5 W/mK.

Solution

For Plate :

Characteristic Length $L_c = \frac{L}{2}$

$$= \frac{0.05}{2}$$

$$\boxed{L_c = 0.025 \text{ m}}$$

We know,

Biot number $B_i = \frac{hL_c}{K}$

$$= \frac{285 \times 0.025}{42.5}$$

$$\Rightarrow \boxed{B_i = 0.1675}$$

0.1 < B_i< 100, So this is infinite solid type problem.

Infinite Solids

Case (i)

[To calculate centre line temperature (or) Mid plane temperature for infinite plate, refer HMT data book Page No.59 Heisler chart].

$$\text{X axis} \rightarrow \text{Fourier number} = \frac{\alpha t}{L_c^2}$$

$$= \frac{1.19 \times 10^{-5} \times 180}{(0.025)^2}$$

$$\boxed{\text{X axis} \rightarrow \text{Fourier number} = 3.42}$$

$$\text{Curve} = \frac{hL_c}{K}$$

$$= \frac{285 \times 0.025}{42.5} = 0.167$$

$$\boxed{\text{Curve} = \frac{hL_c}{K} = 0.167}$$

X axis value is 3.42, curve value is 0.167, corresponding Y axis value is 0.64

$$\text{Y axis} = \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.64$$

$$\frac{T_0 - T_\infty}{T_i - T_\infty} = 0.64$$

$$\Rightarrow \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.64$$

$$\Rightarrow \quad \frac{T_0 - 333}{673 - 333} = 0.64$$

$$\Rightarrow \quad \boxed{T_0 = 550.6 \text{ K}}$$

$$\boxed{\text{Center line temperature } T_0 = 550.6 \text{ K}}$$

Case (ii) Temperature (T_x) at a distance of 0.0125 m from mid plane

[Refer HMT data book Page No.60, Heisler chart]

$$X \text{ axis} \rightarrow \text{Biot number } B_i = \frac{hL_c}{K} = 0.167$$

$$\text{Curve} \rightarrow \frac{x}{L_c} = \frac{0.0125}{0.025} = 0.5$$

X axis value is 0.167, curve value is 0.5, corresponding Y axis value is 0.97.

$$\frac{T_x - T_\infty}{T_0 - T_\infty} = 0.97$$

$$Y \text{ axis} = \frac{T_x - T_\infty}{T_0 - T_\infty} = 0.97$$

$$\Rightarrow \quad \frac{T_x - T_\infty}{T_0 - T_\infty} = 0.97$$

$$\Rightarrow \quad \frac{T_x - 333}{550.6 - 333} = 0.97$$

$$\Rightarrow \quad \boxed{T_x = 544 \text{ K}}$$

Temperature inside the plate 1.25 cm from the mid plane is 544 K.

Review questions:-

1. State Fourier's law of heat conduction. **(May/June 2013, Nov/Dec 2013, April/May 2011 Nov/Dec 2014) (Ref.pg: 2, Qn. no: 1)**

2. Define fin efficiency and fin effectiveness. **(May/June 2013, Nov/Dec 2010, Nov/Dec 2014) (Ref.pg: 2, Qn. no: 2)**

3. What is lumped system analysis? When is it used**? (May/June 2013, April/May 2011, Nov/Dec 2010) (Ref.pg: 3, Qn. no: 3)**

4. Write the three dimensional heat transfer Poisson's and Laplace equations in Cartesian co-ordinates**.** **(May/June-2012) (Ref.pg: 3, Qn. no: 4)**

5. A 3 mm wire of thermal conductivity 19 W/mK at a steady heat generation of 500 MW/m^3. Determine the centre temperature if the outside temperature is maintained at 25°C. h = 4500W/m^2**K** **(May/June 2012) (Ref.pg: 3, Qn. no: 5)**

6. What are the two mechanisms of heat conduction in solids? **(Nov/Dec 2011) (Ref.pg: 4, Qn. no: 6)**

7. What is the purpose of attaching fins to a surface? What are the different types of fin profiles? **(Nov/Dec 2011 (Ref.pg: 4, Qn. no: 7)**

8. In what medium, the lumped system analysis is more likely to be applicable? An aluminium or wood? Why? **(Nov/Dec 2011) (Ref.pg: 4, Qn. no: 8)**

9. What is heat generation in solids? Give examples.**(April/May 2011) (Ref.pg: 4, Qn. no: 9)**

10. Discuss the mechanism of heat conduction in solids.**(May/June 2009) (Ref.pg: 5, Qn. no: 10)**

11. What is the physical meaning of Fourier number?**(May/June 2009) (Ref.pg: 5, Qn. no: 11)**

12. A temperature difference of 500°C is applied across a fire-clay brick, 10cm thick having a thermal conductivity of 1 W/mK. Find the heat transfer rate per unit area. (Apr/May2008) **(Ref.pg: 5, Qn. no: 12)**

13. Write the general 3-D heat conduction equation in cylindrical co-ordinates.

(Apr/May2008) (Ref.pg: 5, Qn. no: 13)

14. Biot number is the ratio between .... and **..........(Apr/May 2008) (Ref.pg: 5, Qn. no:**
15. What is the main advantage of parabolic fins? **(Nov/Dec 2007) (Ref.pg: 5, Qn. no: 15)**

16. What is sensitivity of a thermocouple**? (Nov/Dec 2007)(Ref.pg: 5, Qn. no: 16)**

17.Define critical radius of insulation**. (Nov/Dec 2007) (Ref.pg: 6, Qn. no: 17)**

18. Mention the importance of Biot number. **(Nov/Dec 2007) (Ref.pg: 6, Qn. no: 18)**

19. What is use of Heislers chart?**(May/June 2007) (Ref.pg: 6, Qn. no:20)**

20. Write any two examples of heat conduction with heat generation **(May/June 2014):**

Some examples of heat generation are resistance heating in wires, exothermic chemical reactions in solids, and nuclear reaction

21. Define critical thickness of insulation with its significance. **(May/June 2014) (Ref.pg: 11, Qn. no: 50)**

22. State Fourier's law of heat conduction. **(Nov/Dec 2014) (Ref.pg: 2, Qn. no: 1)**

23. Define fin efficiency and fin effectiveness. **(Nov/Dec 2014) (Ref.pg: 2, Qn. no: 2)**

Part-B

1. a) Explain the mechanism of heat conduction in solids: **(May/June-2013, Nov/Dec 2014)(Ref.pg: 5, Qn. No: 10)**

b) At a certain instant of time, temperature distribution in a long cylindrical tube is T = 800 +100 r- 5000 r^2 where, T is in ^oC and r in mm. The inner and outer radii of the tube are respectively 30 cm and 50 cm. the tube material has a thermal conductivity of 58 W/m.K and a thermal diffusivity of 0.004 m^2/hr. Determine the rate of heat flow at inside and outside surfaces per unit length, rate of heat storage per unit length and rate of change of temperature at inner and outer surfaces**.** : **(May/June-2013)(Ref.pg: 10, Qn. no: 1)**

2. (i) Explain different fin profiles: **(May/June-2013)(Ref.pg: 4, Qn. no:7)**

ii) Circumferential rectangular fins of 140mm wide and 5mm thick are fitted on a 200mm diameter tube. The fin base temperature is 170^oC and the ambient temperature and the ambient temperature is 25^oC. Estimate fin efficiency and heat loss per fin.

Take: Thermal conductivity, k = 220 W/mK.

Heat transfer co-efficient, h = 140 W/m^2K**(May/June-2013)(Ref.pg: 12, Qn. no:2)**

3.A furnace wall is made up of three layer thickness 25cm, 10cm, and 15cm with thermal conductivities of 1.65w/mk and 9.2 w/mk respectively .the inside is exposed to the gasses at 1250^{0}c with is convection coefficient of 25 w/m^{20}c and inside surface of 1100^{0}c ,the outside surface is exposed to the air at 25^{0}c with convection coefficient of 12 w/m^2K .determine (1)the unknown thermal conductivity (2) THE overall heat transfer coefficient (3) ALL surface temperature **[May/June-12] (Ref.pg: 14, Qn. no:3)**

4. Pin fins Aare provided to increase the heat transfer rate from hot surface .which of the following arrange will given higher heat transfer rate ?(1) 6 fins of 10 cm length (2) 12 fins of 5cm length .take K of fin material =200 w/mk and h =20w/m^{20}c cross sectional area of the fins =2cm^2,perimeter of fin =4cm ,find the base temperature =230^{0}c, surrounding air temperature =300^{0}c **[May /June 12] (Ref.pg: 15, Qn. no:4)**

5. A composite wall consists of 2.5 cm thick copper plate, a 3.2 cm layer of asbestos insulation and a5cm layer fiber plate .thermal conductivities off the material are respectively 355,0.110 and 0.0489 w/mk. The temperature difference across the composite wall is 560^{0}c the side and ^{0}c on the other side. The find the heat flow through the wall per unit area and the interface temp .between asbestos and fiber plate. **[Nov/Dec-12](Ref.pg: 16, Qn. no:5)**

6. The cylinder of a 2-stroke SI engine is constructed of aluminum alloy (K=186 w/mk).The height and outside diameter of the cylinder are respectively 15cm and 5cm.understand operating condition ,the outer surface the cylinder is at500k an is exposed to the ambient air at 3000K ,with a convention heat transfer coefficient of50 w/m^{2}K equally spaced annular fins are attached with cylinder to increase the heat transfer .there are five such fins with uniform thickness ,t=6mm and the length ,l=20mm. calculate the increase in heat transfer due to the addition fins **[Nov/Dec-11] (Ref.pg: 16, Qn. no:6)**

7. A cold storage room has walls made of 23cm of bricks on the outsie,8cm of plastic foam and finally 1.5cm of wood on the inside .the outside and inside air temperature are22 and-2 respectively. the inside and outside heat transfer coefficient are respectively 29 and 12 w/m^{2}k .the thermal conductivities of brick ,foam and wood are 0.98,0.02 and0.12 w/mk respectively .if the total wall area is 90m/t determine the rate of heat removal by refrigerator and the temperature of the inside surface of the brick **[April/May-11] (Ref.pg:18, Qn. no:7)**

8. A steel rod of diameter 112mm and 60mm long with insulated end that has a thermal conductivity of 32w/m^{0}c is to be used as a spine .it is expressed to surrounding with a temperature at 60^{0}c and heat transfer coefficient of 55w/m^2 .the temperature the base of the fin is 95^{0}c .calculate the fin efficiency,the temperature at the edge of the spine and the heat dissipation **[Nov/Dec 10] (Ref.pg: 19, Qn. no:8)**

9. a) Two slabs each of 120mm thick have thermal conductivities of 14 w/m and 210 w/m .These are placed in contact but due to roughness only 30 of area placed in contact and gap in the remaining area is 0.025mm thick and is filled with air .If the temperature of the face of the hot surface is at 220 and the outside surface of the other slab is at 30 ,calculate the heat flow through the composite system .Assume that conductivity of the air is 0.032 and the half of the contact (of the contact area)is due to either metal **[Nov/Dec 10] (Ref.pg: 20, Qn. no:9)**

10. A 60 mm thick large steel plate [K=42.6w/m^{0}c,X=0.043m^2/h] initially at 440^{0}c is suddenly exposed on the both side to an ambient with convection heat transfer coefficient 235w/m^{20}c and temperature inside the plate 15mm from the mid plane after 4.3minutes **[Nov/Dec 10] (Ref.pg: 20, Qn. no:10)**

11. Obtain an expression for the general heat conduction equation in cartesian coordinates. **[Nov/Dec 2006]** **(Ref.pg: 23, Qn. no: 13)**

12. a) An exterior wall of a house is covered by 10mm common bricks (K=0.7w/m k) followed by a4cm layer of gypsum plaster (K=0.48w/m k) .what thickness of loosely packed insulation (K=0.065w/m k)should be added to reduce the heat loss through the wall by 80%? **[May-2004]** **(Ref.pg: 26, Qn. no: 14)**

13. A plane wall 10cm thick generator heat at rate of $4*10^4 wm^3$ when a electric current is passed through it. the convective heat transfer coefficient between each face of the wall and ambient air is50 w/m^3.determine (a) surface temperature (b) the maximum air temperature the wall assume that ambient air temperature to be 20^0c and the thermal conductivity of the wall material to be 15 w/m k **[April- 98] (Ref.pg: 27, Qn. no:15)**

14.Derive the general heat conduction equation in cylindrical coordinate system **(May/June 2014)**

Now consider a thin cylindrical shell element of thickness Δr in a long cylinder, as shown in Fig. 2–14. Assume the density of the cylinder is ρ, the specific heat is c, and the length is L. The area of the cylinder normal to the direction of heat transfer at any location is $A = 2\pi rL$ where r is the value of the radius at that location. Note that the heat transfer area A depends on r in this case, and thus it varies with location. An *energy balance* on this thin cylindrical shell element during a small time interval Δt can be expressed as

$$\begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r\end{pmatrix} - \begin{pmatrix}\text{Rate of heat}\\ \text{conduction}\\ \text{at } r+\Delta r\end{pmatrix} + \begin{pmatrix}\text{Rate of heat}\\ \text{generation}\\ \text{inside the}\\ \text{element}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{content of the}\\ \text{element}\end{pmatrix}$$

or

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{E}_{\text{gen, element}} = \frac{\Delta E_{\text{element}}}{\Delta t} \qquad 1$$

The change in the energy content of the element and the rate of heat generation within the element can be expressed as

$$\Delta E_{\text{element}} = E_{t+\Delta t} - E_t = mc(T_{t+\Delta t} - T_t) = \rho c A \Delta r (T_{t+\Delta t} - T_t) \qquad 2$$

$$\dot{E}_{\text{gen, element}} = \dot{e}_{\text{gen}} V_{\text{element}} = \dot{e}_{\text{gen}} A \Delta r \qquad 3$$

Substituting into Eq. 1 , we get

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{e}_{\text{gen}} A \Delta r = \rho c A \Delta r \frac{T_{t+\Delta t} - T_t}{\Delta t} \qquad 4$$

where $A = 2\pi rL$. You may be tempted to express the area at the *middle* of the element using the *average* radius as $A = 2\pi(r + \Delta r/2)L$. But there is nothing we can gain from this complication since later in the analysis we will take the limit as $\Delta r \to 0$ and thus the term $\Delta r/2$ will drop out. Now dividing the equation above by $A\Delta r$ gives

$$-\frac{1}{A}\frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} + \dot{e}_{\text{gen}} = \rho c \frac{T_{t+\Delta t} - T_t}{\Delta t} \qquad 5$$

Taking the limit as $\Delta r \to 0$ and $\Delta t \to 0$ yields

$$\frac{1}{A}\frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right) + \dot{e}_{\text{gen}} = \rho c \frac{\partial T}{\partial t} \qquad 6$$

since, from the definition of the derivative and Fourier's law of heat conduction,

$$\lim_{\Delta r \to 0} \frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r}\left(-kA\frac{\partial T}{\partial r}\right) \qquad 7$$

Noting that the heat transfer area in this case is $A = 2\pi rL$, the one-dimensional transient heat conduction equation in a cylinder becomes

Variable conductivity:
$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \dot{e}_{\text{gen}} = \rho c\frac{\partial T}{\partial t} \qquad 8$$

For the case of constant thermal conductivity, the previous equation reduces to

Constant conductivity:
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{\text{gen}}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad 9$$

where again the property $\alpha = k/\rho c$ is the thermal diffusivity of the material. Eq. 9 reduces to the following forms under specified conditions (Fig. 2–15):

(1) *Steady-state:* $(\partial/\partial t = 0)$
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{e}_{\text{gen}}}{k} = 0 \qquad 10$$

(2) *Transient, no heat generation:* $(\dot{e}_{\text{gen}} = 0)$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad 11$$

(3) *Steady-state, no heat generation:* $(\partial/\partial t = 0$ and $\dot{e}_{\text{gen}} = 0)$
$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0 \qquad 12$$

Note that we again replaced the partial derivatives by ordinary derivatives in the one-dimensional steady heat conduction case since the partial and ordinary derivatives of a function are identical when the function depends on a single variable only [$T = T(r)$ in this case].