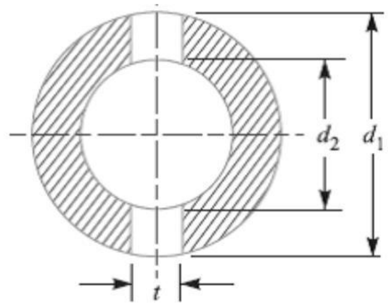
From this equation, the induced crushing stress may be checked.

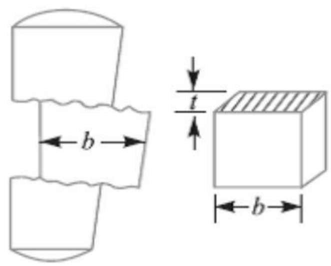
**4. Failure of the socket in tension across the slot**




$$P = \left\{\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\,t\right\}\sigma_t$$

From this equation, outside diameter of socket ($d_1$) may be determined.

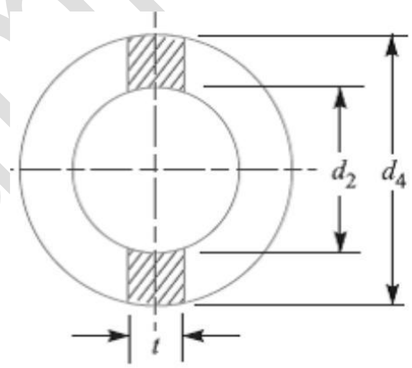
**5. Failure of cotter in shear**




$$P -2\,b \times t \times \tau$$

From this equation, width of cotter (b) is determined.

**6. Failure of the socket collar in crushing**




$$P = (d_4 - d_2)\,t \times \sigma_c$$

From this equation, the diameter of socket collar ($d_4$) may be obtained.

**7. Failure of socket end in shearing**

$$P = 2\,(d_4 - d_2)\,c \times \tau$$

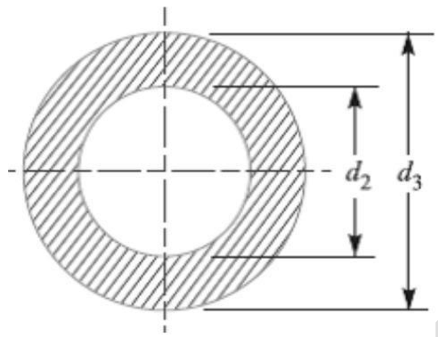
From this equation, the thicknesss of socket collar (c) may be obtained.

**8. Failure of rod end in shear**

$$P = 2\,a \times d_2 \times \tau$$

From this equation, the distance from the end of the slot to the end of the rod (a) may be obtained.

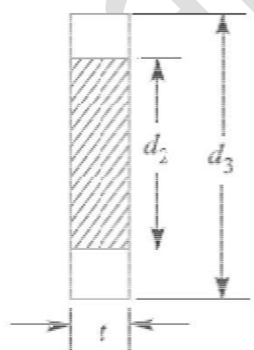
**9. Failure of spigot collar in crushing**




$$P = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c$$

From this equation, the diameter of the spigot collar ($d_3$) may be obtained.

**10. Failure of the spigot collar in shearing**

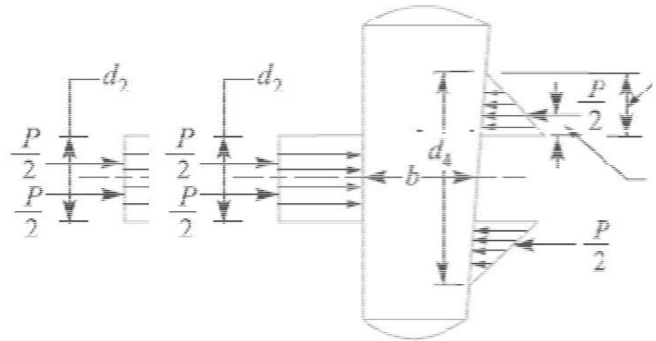



$$P = \pi\, d_2 \times t_1 \times \tau$$

From this equation, the thicknesss of spigot collar ($t_1$) may be obtained.

**11. Failure of cotter in bending**

The maximum bending moment occurs at the centre of the cotter and is given by




$$M_{max} = \frac{P}{2}\left(\frac{1}{3} \times \frac{d_4 - d_2}{2} + \frac{d_2}{2}\right) - \frac{P}{2} \times \frac{d_2}{4}$$

$$= \frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{2} - \frac{d_2}{4}\right) = \frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{4}\right)$$

We know that section modulus of the cotter,

$$Z = t \times b^2 / 6$$

Bending stress induced in the cotter,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{\frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{4}\right)}{t \times b^2 / 6} = \frac{P\,(d_4 + 0.5\,d_2)}{2\,t \times b^2}$$

This bending stress induced in the cotter should be less than the allowable bending stress of the cotter.

**11.** The length of cotter (l) in taken as 4 d.

**12.** The taper in cotter should no t exceed 1 in 24. In case the greater taper is required, then a locking device must be provided.

**13.** The draw of cotter is generally taken as 2 to 3 mm.

**Notes: 1.** when all the parts of the joint are made of steel, the following proportions in terms of diameter of the rod (d) are gennerally adopted:

$d_1$ = 1.75 d , $d_2$ = 1.21 d , $d_3$ = 1.5 d , $d_4$ = 2.4 d , a = c = 0.75 d , b = 1.3 d, l = 4 d , t = 0.31 d ,$t_1$ = 0.45 d , e = 1.2 d.

Taper of cotter = 1 in 25, and dra w of cotter = 2 to 3 mm.

**6. If the rod and cotter are made of steel or wrought iron, then $\tau = 0.8\, \sigma_t$ and $\sigma_c = 2\, \sigma_t$** may be taken.

**References:**

12.Machine Design - V.Bandari .

13.Machine Design – R.S. Khurm i

14.Design Data hand Book - S MD Jalaludin.

Problem:

Design and draw a cotter joint to support a load varying from 30 kN in compression to 30 kN in tension. The material used is carbon steel for which the following allowable stresses may be used. The load is applied stattically. Tensile stress = compressive stress = 500 MPa ; shear stress = 35 MPa and crushing stress = 90 MPa.

**Solution.** Given : $P = 30\text{ kN} = 30 \times 10^3\text{ N}$ ; $\sigma_t = 50\text{ MPa} = 50\text{ N/mm}^2$ ; $\tau = 35\text{ MPa} = 35\text{ N/mm}^2$ ; $\sigma_c = 90\text{ MPa} = 90\text{ N/mm}^2$

1. *Diameter of the rods*

Let $d$ = Diameter of the rods.

Considering the failure of the rod in tension. We know that load ($P$),

$$30 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 50 = 39.3\, d^2$$

$$\therefore \quad d^2 = 30 \times 10^3 / 39.3 = 763 \quad \text{or} \quad d = 27.6 \text{ say } 28 \text{ mm} \quad \textbf{Ans.}$$

2. *Diameter of spigot and thickness of cotter*

Let $d_2$ = Diameter of spigot or inside diameter of socket, and

$t$ = Thickness of cotter. It may be taken as $d_2 / 4$.

Considering the failure of spigot in tension across the weakest section. We know that load ($P$),

$$30 \times 10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times \frac{d_2}{4}\right]50 = 26.8\,(d_2)^2$$

$$\therefore \quad (d_2)^2 = 30 \times 10^3 / 26.8 = 1119.4 \quad \text{or} \quad d_2 = 33.4 \text{ say } 34 \text{ mm}$$

and thickness of cotter, $t = \dfrac{d_2}{4} = \dfrac{34}{4} = 8.5\text{ mm}$

Let us now check the induced crushing stress. We know that load ($P$),

$$30 \times 10^3 = d_2 \times t \times \sigma_c = 34 \times 8.5 \times \sigma_c = 289\,\sigma_c$$

$$\therefore \quad \sigma_c = 30 \times 10^3 / 289 = 103.8\text{ N/mm}^2$$

Since this value of $\sigma_c$ is more than the given value of $\sigma_c = 90\text{ N/mm}^2$, therefore the dimensions $d_2 = 34$ mm and $t = 8.5$ mm are not safe. Now let us find the values of $d_2$ and $t$ by substituting the value of $\sigma_c = 90\text{ N/mm}^2$ in the above expression, *i.e.*

$$30 \times 10^3 = d_2 \times \frac{d_2}{4} \times 90 = 22.5\,(d_2)^2$$

$$\therefore \quad (d_2)^2 = 30 \times 10^3 / 22.5 = 1333 \quad \text{or} \quad d_2 = 36.5 \text{ say } 40 \text{ mm} \quad \textbf{Ans.}$$

and $t = d_2 / 4 = 40 / 4 = 10\text{ mm}$ **Ans.**

3. *Outside diameter of socket*

Let $d_1$ = Outside diameter of socket.

Considering the failure of the socket in tension across the slot. We know that load ($P$),

$$30 \times 10^3 = \left[\frac{\pi}{4}\left\{(d_1)^2 - (d_2)^2\right\} - (d_1 - d_2)\,t\right]\sigma_t$$

$$= \left[\frac{\pi}{4}\left\{(d_1)^2 - (40)^2\right\} - (d_1 - 40)\,10\right]50$$

$$30 \times 10^3/50 = 0.7854\,(d_1)^2 - 1256.6 - 10\,d_1 + 400$$

or $(d_1)^2 - 12.7\,d_1 - 1854.6 = 0$

$$\therefore \quad d_1 = \frac{12.7 \pm \sqrt{(12.7)^2 + 4 \times 1854.6}}{2} = \frac{12.7 \pm 87.1}{2}$$

$= 49.9$ say 50 mm Ans. ...(Taking +ve sign)

4. *Width of cotter*

Let $b$ = Width of cotter.

Considering the failure of the cotter in shear. Since the cotter is in double shear, therefore load ($P$),

$$30 \times 10^3 = 2\,b \times t \times \tau = 2\,b \times 10 \times 35 = 700\,b$$

$$\therefore \quad b = 30 \times 10^3 / 700 = 43 \text{ mm Ans.}$$

5. *Diameter of socket collar*

Let $d_4$ = Diameter of socket collar.

Considering the failure of the socket collar and cotter in crushing. We know that load ($P$),

$$30 \times 10^3 = (d_4 - d_2)\,t \times \sigma_c = (d_4 - 40)10 \times 90 = (d_4 - 40)\,900$$

$$\therefore \quad d_4 - 40 = 30 \times 10^3 / 900 = 33.3 \text{ or } d_4 = 33.3 + 40 = 73.3 \text{ say 75 mm Ans.}$$

6. *Thickness of socket collar*

Let $c$ = Thickness of socket collar.

Considering the failure of the socket end in shearing. Since the socket end is in double shear, therefore load ($P$),

$$30 \times 10^3 = 2(d_4 - d_2)\,c \times \tau = 2\,(75 - 40)\,c \times 35 = 2450\,c$$

$$\therefore \quad c = 30 \times 10^3 / 2450 = 12 \text{ mm Ans.}$$

7. *Distance from the end of the slot to the end of the rod*

Let $a$ = Distance from the end of slot to the end of the rod.

Considering the failure of the rod end in shear. Since the rod end is in double shear, therefore load ($P$),

$$30 \times 10^3 = 2\,a \times d_2 \times \tau = 2a \times 40 \times 35 = 2800\,a$$

$$\therefore \quad a = 30 \times 10^3 / 2800 = 10.7 \text{ say 11 mm Ans.}$$

8. *Diameter of spigot collar*

Let $d_3$ = Diameter of spigot collar.

Considering the failure of spigot collar in crushing. We know that load ($P$),

$$30 \times 10^3 = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c = \frac{\pi}{4}\left[(d_3)^2 - (40)^2\right]90$$

or
$$(d_3)^2 - (40)^2 = \frac{30 \times 10^3 \times 4}{90 \times \pi} = 424$$

$$\therefore \quad (d_3)^2 = 424 + (40)^2 = 2024 \text{ or } d_3 = 45 \text{ mm Ans.}$$

9. *Thickness of spigot collar*

Let $t_1$ = Thickness of spigot collar.

Considering the failure of spigot collar in shearing. We know that load ($P$),

$$30 \times 10^3 = \pi d_2 \times t_1 \times \tau = \pi \times 40 \times t_1 \times 35 = 4400 t_1$$

$\therefore$ $t_1 = 30 \times 10^3 / 4400 = 6.8$ say 8 mm Ans.

10. The length of cotter ($l$) is taken as 4 $d$.

$\therefore$ $l = 4d = 4 \times 28 = 112$ mm Ans.

11. The dimension $e$ is taken as 1.2 $d$.

$\therefore$ $e = 1.2 \times 28 = 33.6$ say 34 mm Ans.

**REFERENCES:**

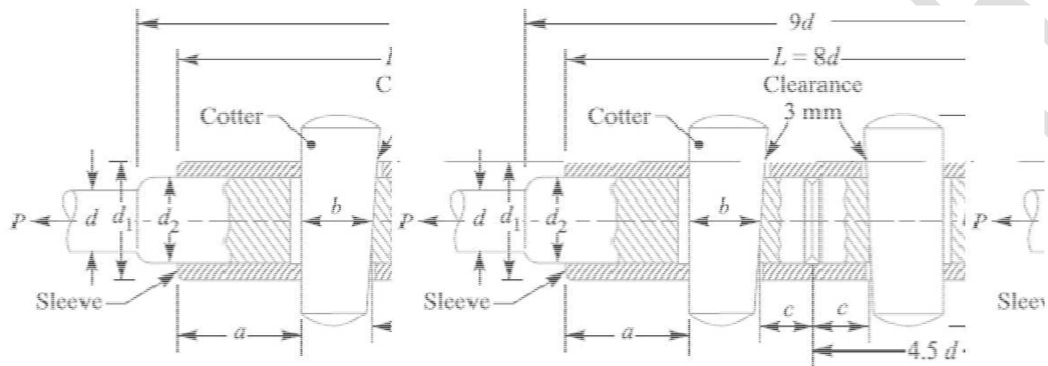

8. Machine Design - V.Bandari .

9. Machine Design – R.S. Khurm i

10.Design Data hand Book - S MD Jalaludin.

**Sleeve and Cotter Joint**

Sometimes, a sleeve and cotter joint as shown in Fig., is used to connect two round rods or bars. In this type of joint, a sleeve or muff is used over the two rods and then two cotters (one on each rod end) are inserted in the holes provided for them in the sleeve and roods. The taper of cotter is usually 1 in 24. It maay be noted that the taper sides of the two cotters should face each other as shown in Fig. The clearance is so adjusted that when the cotters are driven in, the two rods come closer to each other thus making the joint tight.




The various proportions for the sleeve and cotter joint in terms of the diameter of rod (d ) are as follows :

Outside diameter of sleeve,

$d_1 = 2.5$ d

Diameter of enlarged end of rod,

$d_2$ = Inside diameter of sleeve = 1.25 d

Length of sleeve, L = 8 d

Thickness of cotter, t = d2/4 or 0.31 d

Width of cotter, b = 1.25 d

Length of cotter, l = 4 d

Distance of the rod end (a) fro m the beginning to the cotter hole (inside the sleeve end) = Distance of the rod end (c) from its end to the cotter hole = 1.25 d

**Design of Sleeve and Cotter Joint**

The sleeve and cotter joint is shown in Fig.

Let P = Load carried by the rods,

d = Diameter of the rods,

$d_1$ = Outside diameter of sleeve,

$d_2$ = Diameter of the enlarged end of rod,

t = Thickness of cotter,

l = Length of cotter,

b = Width of cotter,

a = Distance of the rod end from the beginning to the cotter hole (inside the sleeve end),

c = Distance of the rod end from its end to the cotter hole,

**$\sigma_t$ , $\tau$ and $\sigma_c$** = Permissible tensile, shear and crushing stresses respectively for the material of the rods and cotter.

The dimensions for a sleeve and cotter joint may be obtained by considering the various modes of failure as discussed below:

**1. Failure of the rods in tension**

The rods may fail in tension due to the tensile load P. We know that

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rods (d) may be obtained.

**2. Failure of the rod in tension across the weakest section (i.e. slot)**

$$P = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t$$

From this equation, the diameter of enlarged end of the rod ($d_2$) may be obtained. The thickness of cotter is usually taken as $d_2$ / 4.

**3. Failure of the rod or cotter in crushing**

$$P = d_2 \times t \times \sigma_c$$

From this equation, the induced crushing stress may be checked.

**4. Failure of sleeve in tension across the slot**

$$P = \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\,t\right]\sigma_t$$

From this equation, the outside diameter of sleeve ($d_1$) may be obtained.

**5. Failure of cotter in shear**

$$P = 2b \times t \times \tau$$

From this equation, width of cotter (b) may be determined.

**6. Failure of rod end in shear**

$$P = 2\,a \times d_2 \times \tau$$

From this equation, distance (a) may be determined.

**7. Failure of sleeve end in shear**

$$P = 2\,(d_1 - d_2)\,c \times \tau$$

From this equation, distance (c) may be determined.



## UNIT 4
## SPRINGS

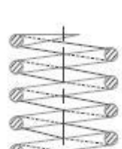
**Introduction**

A spring is defined as an elast ic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows:

1. To cushion, absorb or control energy due to either shock or vibration as i n car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.
2. To apply forces, as in brakes, clutches and spring loaded valves.
3. To control motion by maintain ing contact between two elements as in cams a nd followers.
4. To measure forces, as in spring balances and engine indicators.
5. To store energy, as in watches, toys, etc.

**Types of springs:**

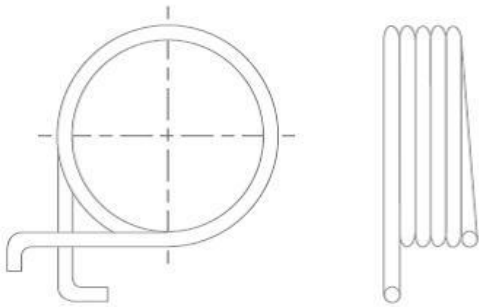
**1. Helical springs.** The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads.



**2. Conical and volute springs.** The conical and volute springs, as shown in Fig. 23.2, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired

(*a*) Conical spring. (*b*) Volute spring.

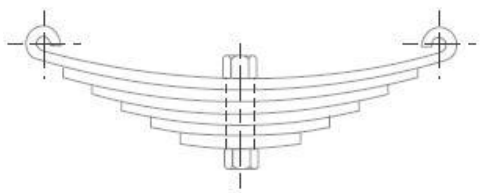
**3. Torsion springs.** These spri ngs may be of **helical** or **spiral** type as show n in Fig. The **helical type** may be used only i n applications where the load tends to wind up the spring and are used in various electrical me chanisms.



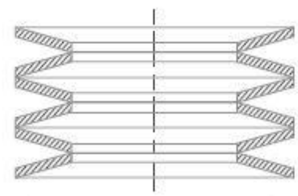
(*a*) Helical torsion spring.

(*b*) Spiral torsion spring.

**4. Laminated or leaf springs.** The laminated or leaf spring (also known as **flat spring** or **carriage spring**) consists of a n umber of flat plates (known as leaves) of varyin g lengths held together by means of clamps and bolts.



Laminated or leaf springs.



Disc or bellevile springs.

**5. Disc or bellevile springs.** Th ese springs consist of a number of conical discs held together against slipping by a central bolt or tube.

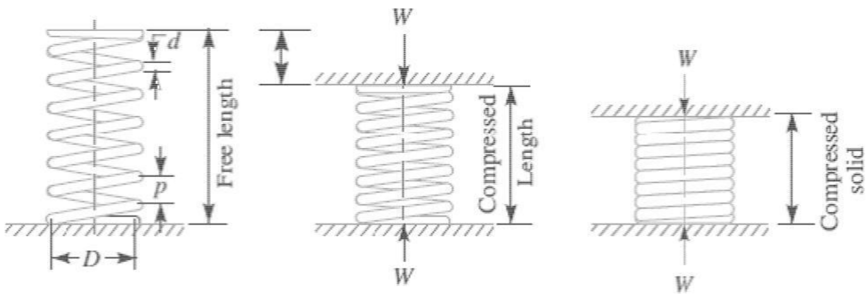
**6. Special purpose springs.** These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or li quid) can behave as a compression spring. These springs are used for special types of application only.

**<u>Terms used in Compression Sp rings</u>**

**1. Solid length.** When the comp ression spring is compressed until the coils come in contact with each other, then the spring is said to be **solid.**

Solid length of the spring, $L_S = n'.d$ where n' = Total number of coils, and d = Diameter of the wire.

**2. Free length.** The free length of a compression spring, as shown in Fig., is the length of the spring in the free or unloaded condition.




Free length of the spring,

$L_F$ = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$$= n'.d + \delta_{max} + 0.15\,\delta_{max}$$

**3. Spring index.** The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Spring index, C = D / d where D = Mean diameter of the coil, and d = Diameter of the wire.

**4. Spring rate.** The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically, Spring rate, k = W / δ where W = Load, and δ = Deflection of the spring.

**5. Pitch.** The pitch of the coil is defined as the axial distance between adj acent coils in uncompressed state. Mathematically, Pitch of the coil,

$$p = \frac{\text{Free Length}}{n' - 1}$$

**<u>Stresses in Helical Springs of C ircular Wire</u>**

Consider a helical compression spring made of circular wire and subjected to an axial load *W*, as shown in Fig.(*a*).

Let *D* = Mean diameter of th e spring coil,

*d* = Diameter of the spring wire,

*n* = Number of active coils,

*G* = Modulus of rigidity f or the spring material,

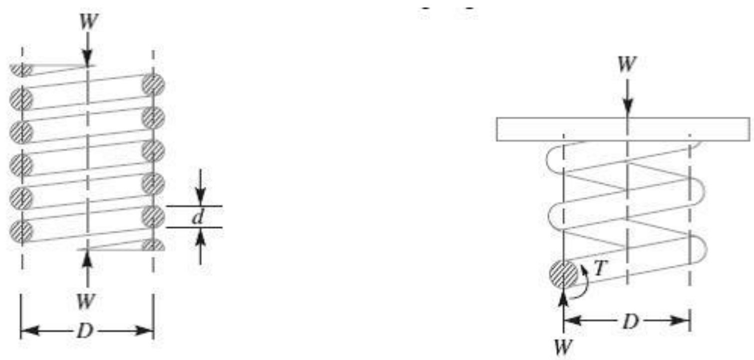
*W* = Axial load on the sp ring,

τ = Maximum shear stress induced in the wire,

*C* = Spring index = *D*/*d*,

*p* = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load *W*.




(a) Axially loaded helical spring. (b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Now consider a part of the compression spring as shown in Fig. (*b*). The load *W* tends to rotate the wire due to the twistin g moment ( *T* ) set up in the wire. Thus torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig.(*b*), is in equilibrium under the action of two forces *W* and the twisting moment *T*. We know that the twisting moment,

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8 W.D}{\pi d^3} \qquad \ldots(i)$$

The torsional shear stress diagra m is shown in Fig. (*a*).

In addition to the torsional shear stress ($\tau_1$) induced in the wire, the following stresses also act on the wire:

**1.** Direct shear stress due to the load *W*, and

**2.** Stress due to curvature of wire .

We know that the resultant shear stress induced in the wire,

$$\tau = \tau_1 \pm \tau_2 = \frac{8 W.D}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

Maximum shear stress induced i n the wire,

= Torsional shear stress + Direct shear stress

$$= \frac{8 W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8 W.D}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8\,W.D}{\pi\,d^3} \qquad \text{...(iii)}$$

... (Substituting $D/d = C$)

where $K_S$ = Shear stress factor $= 1 + \frac{1}{2C}$

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} = K \times \frac{8\,W.C}{\pi\,d^2} \qquad \text{...(iv)}$$

where $K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$

**Deflection of Helical Springs o f Circular Wire**

Total active length of the wire,

$l$ = Length of one coil × No. of active coils = $\pi\,D \times n$

Let $\theta$ = Angular deflection of the wire when acted upon by the torque $T$.

∴ Axial deflection of the spring,

$$\delta = \theta \times D/2 \qquad \text{...(i)}$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

$$\therefore \quad \theta = \frac{T.l}{J.G} \qquad \text{...}\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where $J$ = Polar moment of inertia of the spring wire

$= \frac{\pi}{32} \times d^4$, $d$ being the diameter of spring wire.

and $G$ = Modulus of rigidity for the material of the spring wire.

Now substituting the values of $l$ and $J$ in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi\,D.n}{\frac{\pi}{32} \times d^4\,G} = \frac{16W.D^2.n}{G.d^4} \qquad \text{...(ii)}$$

Substituting this value of $\theta$ in equation (*i*), we have

$$\delta = \frac{16W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8\,W.D^3.n}{G.d^4} = \frac{8\,W.C^3.n}{G.d} \qquad \text{... } (\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n} = \frac{G.d}{8\,C^3.n} = \text{constant}$$

**Buckling of Compression Spri ngs**

It has been found experimentally that when the free length of the spring ($L_F$) is more than four times the mean or pitch dia meter (D), then the spring behaves like a column and may fail by buckling at a comparatively l ow load.

$$W_{cr} = k \times K_B \times L_F$$

where k = Spring rate or stiffness of the spring = W/δ,

$L_F$ = Free length of the spring, and

$K_B$ = Buckling factor depending upon the ratio $L_F$ / D.

**Surge in springs**

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.

$$f_n = \frac{d}{2\pi D^2 . n}\sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

Where d = Diameter of the wire,

D = Mean diameter of the spring,

n = Number of active turns,

G = Modulus of rigidity,

g = Acceleration due to gravity, and

ρ = Density of the material of the spring.

**References:**

1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurmi

3. Design Data hand Book - S MD Jalaludin.

Problem: A helical spring is ma e from a wire of 6 mm diameter and has outsid e diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm$^2$, find the axial load which the spring can carry and the deflection per active turn.

**Solution.** Given : $d = 6$ mm ; $D_o = 75$ mm ; $\tau = 350$ MPa $= 350$ N/mm$^2$ ; $G = 84$ kN/mm$^2$ $= 84 \times 10^3$ N/mm$^2$

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

$\therefore$ Spring index, $\quad C = \frac{D}{d} = \frac{69}{6} = 11.5$

Let $\quad W =$ Axial load, and

$\delta / n =$ Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire ($\tau$),

$$350 = K_S \times \frac{8\,W.D}{\pi\,d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

$\therefore \quad W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta - \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm } \textbf{Ans.}$$

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire ($\tau$),

$$350 = K \times \frac{8W.C}{\pi\,d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\,W$$

$\therefore \quad W = 350 / 0.913 = 383.4$ N **Ans.**

and deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm } \textbf{Ans.}$$

Problem: Design a spring for a balance to measure 0 to 1000 N over a scale of le ngth 80 mm. The spring is to be enclosed in a casing of 25 mm diameter. The approxiimate number of turns is 30. The modulus of rigidity is 85 kN/mm$^2$. Also calculate the maximu m shear stress induced.

Solution:

*Design of spring*

Let $D$ = Mean diameter of the spring coil,

$d$ = Diameter of the spring wire, and

$C$ = Spring index = $D/d$.

Since the spring is to be enclosed in a casing of 25 mm diameter, therefore the outer diameter of the spring coil ($D_o = D + d$) should be less than 25 mm.

We know that deflection of the spring (δ),

$$80 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000 \times C^3 \times 30}{85 \times 10^3 \times d} = \frac{240\,C^3}{85\,d}$$

$$\therefore \quad \frac{C^3}{d} = \frac{80 \times 85}{240} = 28.3$$

Let us assume that $d$ = 4 mm. Therefore

$$C^3 = 28.3\,d = 28.3 \times 4 = 113.2 \quad \text{or} \quad C = 4.84$$

and $D = C.d = 4.84 \times 4 = 19.36$ mm **Ans.**

We know that outer diameter of the spring coil,

$$D_o = D + d = 19.36 + 4 = 23.36 \text{ mm Ans.}$$

Since the value of $D_o$ = 23.36 mm is less than the casing diameter of 25 mm, therefore the assumed dimension, $d$ = 4 mm is correct.

*Maximum shear stress induced*

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 4.84 - 1}{4 \times 4.84 - 4} + \frac{0.615}{4.84} = 1.322$$

∴ Maximum shear stress induced,

$$\tau = K \times \frac{8\,W.C}{\pi\,d^2} = 1.322 \times \frac{8 \times 1000 \times 4.84}{\pi \times 4^2}$$

$$= 1018.2 \text{ N/mm}^2 = 1018.2 \text{ MPa Ans.}$$

**References:**

1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurm i

3. Design Data hand Book - S MD Jalaludin.

Problem: Design a helical compr ession spring for a maximum load of 1000 N fo r a deflection of 25 mm using the value of sp ring index as 5. The maximum permissible sh ear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm$^2$.

Take Wahl's factor, $K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$

Solution. Given : $W$ = 1000 N ; $\delta$ = 25 mm ; $C = D/d = 5$ ; $\tau$ = 420 MPa = 420 N/mm$^2$ ; $G$ = 84 kN/mm$^2$ = $84 \times 10^3$ N/mm$^2$

1. *Mean diameter of the spring coil*

Let $D$ = Mean diameter of the spring coil, and

$d$ = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress ($\tau$),

$$420 = K \times \frac{8\,W.C}{\pi\, d^2} = 1.31 \times \frac{8 \times 1000 \times 5}{\pi\, d^2} = \frac{16\,677}{d^2}$$

$\therefore \quad d^2 = 16\,677 / 420 = 39.7 \quad \text{or} \quad d = 6.3 \text{ mm}$

From Table 23.2, we shall take a standard wire of size *SWG* 3 having diameter ($d$) = 6.401 mm.

$\therefore$ Mean diameter of the spring coil,

$D = C.d = 5\,d = 5 \times 6.401 = 32.005$ mm **Ans.** ... ($\because C = D/d = 5$)

and outer diameter of the spring coil,

$D_o = D + d = 32.005 + 6.401 = 38.406$ mm **Ans.**

2. *Number of turns of the coils*

Let $n$ = Number of active turns of the coils.

We know that compression of the spring ($\delta$),

$$25 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 1000\,(5)^3\,n}{84 \times 10^3 \times 6.401} = 1.86\,n$$

$\therefore \quad n = 25 / 1.86 = 13.44$ say 14 **Ans.**

For squared and ground ends, the total number of turns,

$n' = n + 2 = 14 + 2 = 16$ **Ans.**

3. *Free length of the spring*

We know that free length of the spring

$= n'.d + \delta + 0.15\,\delta = 16 \times 6.401 + 25 + 0.15 \times 25$

$= 131.2$ mm **Ans.**

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm} \quad \textbf{Ans.}$$

Problem: Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial de flection of the spring for the load range is 6 m m. Assume a spring index of 5. The permissib le shear stress intensity is 420 MPa and modulus of rigidity, G = 84 kN/mm$^2$. Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

**Solution.** Given : $W_1 = 2250$ N ; $W_2 = 2750$ N ; $\delta = 6$ mm ; $C = D/d = 5$ ; $\tau = 420$ MPa $= 420$ N/mm$^2$ ; $G = 84$ kN/mm$^2 = 84 \times 10^3$ N/mm$^2$

1. *Mean diameter of the spring coil*

Let $D$ = Mean diameter of the spring coil for a maximum load of $W_2 = 2750$ N, and

$d$ = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2750 \times \frac{5d}{2} = 6875\ d \qquad \ldots\left(\because C = \frac{D}{d} = 5\right)$$

We also know that twisting moment ($T$),

$$6875\ d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\ d^3$$

$$\therefore \quad d^2 = 6875 / 82.48 = 83.35 \quad \text{or} \quad d = 9.13 \text{ mm}$$

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter ($d$) = 9.49 mm.

∴ Mean diameter of the spring coil,

$$D = 5d = 5 \times 9.49 = 47.45 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the spring coil,

$$D_o = D + d = 47.45 + 9.49 = 56.94 \text{ mm } \textbf{Ans.}$$

and inner diameter of the spring coil,

$$D_i = D - d = 47.45 - 9.49 = 37.96 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the spring coil*

Let $n$ = Number of active turns.

It is given that the axial deflection ($\delta$) for the load range from 2250 N to 2750 N (*i.e.* for $W$ = 500 N) is 6 mm.

We know that the deflection of the spring ($\delta$),

$$6 = \frac{8\ W.C^3.n}{G.d} = \frac{8 \times 500\ (5)^3\ n}{84 \times 10^3 \times 9.49} = 0.63\ n$$

$$\therefore \quad n = 6 / 0.63 = 9.5 \text{ say } 10 \textbf{ Ans.}$$

For squared and ground ends, the total number of turns,

$$n' = 10 + 2 = 12 \textbf{ Ans.}$$

3. *Free length of the spring*

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{max} = \frac{6}{500} \times 2750 = 33 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\ \delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{152}{12 - 1} = 13.73 \text{ say } 13.8 \text{ mm } \textbf{Ans.}$$

**References:**


1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurm i

3. Design Data hand Book - S MD Jalaludin.

**Energy Stored in Helical Springs of Circular Wire**

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let $W$ = Load applied on the spring, and

$\delta$ = Deflection produced in the spring due to the load $W$.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} \text{ or } W = \frac{\pi\,d^3.\tau}{8\,K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times \pi\,d^3.\tau}{8\,K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\,\tau.D^2.n}{K.d.G}$$

Substituting the values of $W$ and $\delta$ in equation ***(i)***, we have

$$U = \frac{1}{2} \times \frac{\pi\,d^3.\tau}{8\,K.D} \times \frac{\pi\,\tau.D^2.n}{K.d.G}$$

$$= \frac{\tau^2}{4\,K^2.G}(\pi\,D.n)\left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4\,K^2.G} \times V$$

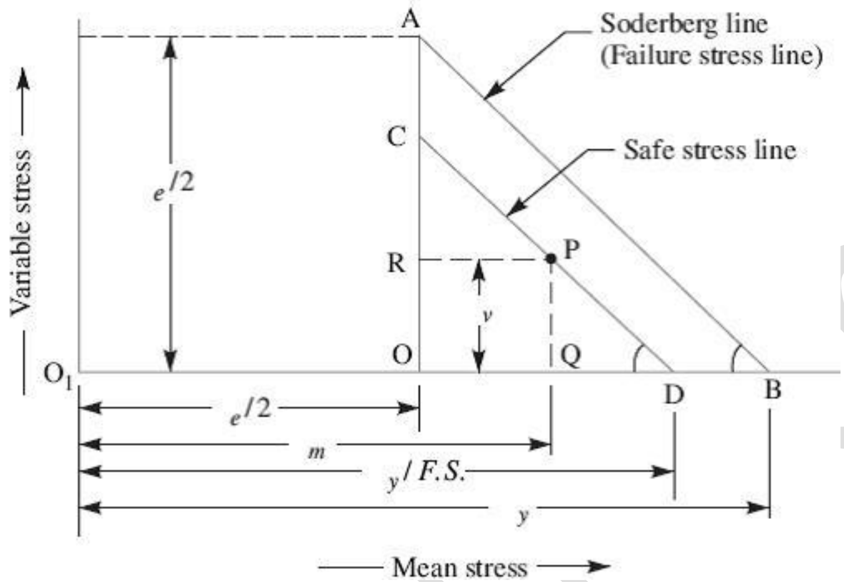
Where $V$ = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

**Helical Springs Subjected to Fatigue Loading**

The helical springs subjected to fatigue loading are designed by using the Soderberg line method. The spring materials are usually tested for torsional endurance strength under a repeated stress that varies from zero to a maximum. Since the springs are ordinarily loaded in one direction only (the load in springs is never reversed in nature), therefore a modified Soderberg diagram is used for springs, as shown in Fig.

The endurance limit for reversed loading is shown at point A where the mean shear stress is equal to $\tau e$ / 2 and the variable shear stress is also equal to $\tau e$ / 2. A line drawn from A to B (the yield point in shear, $\tau y$) gives the Soderberg's failure stress line. If a suitable factor of safety (F.S.) is applied to the yield strength ($\tau y$), a safe stress line CD may be drawn parallel to the line AB, as show n in Fig. Consider a design point P on the line CD. Now the value of factor of safety may be obtained as discussed below:




From similar triangles PQD and AOB, we have

$$\frac{PQ}{QD} = \frac{OA}{OB} \quad \text{or} \quad \frac{PQ}{O_1D - O_1Q} = \frac{OA}{O_1B - O_1O}$$

$$\frac{\tau_v}{\frac{\tau_y}{F.S.} - \tau_m} = \frac{\tau_e / 2}{\tau_y - \frac{\tau_e}{2}} = \frac{\tau_e}{2\tau_y - \tau_e}$$

or

$$2\,\tau_v \,.\, \tau_y - \tau_v .\, \tau_e = \frac{\tau_e \,.\, \tau_y}{F.S.} - \tau_m . \tau_e$$

$$\therefore \quad \frac{\tau_e \,.\, \tau_y}{F.S.} = 2\,\tau_v . \tau_y - \tau_v . \tau_e + \tau_m . \tau_e$$

Dividing both sides by $\tau_e . \tau_y$ and rearranging, we have

$$\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e}$$

**Springs in Series**

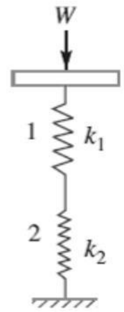
Total deflection of the springs,

$$\delta - \delta_1 + \delta_2$$

$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$k$ = Combined stiffness of the springs.




**Springs in Parallel**

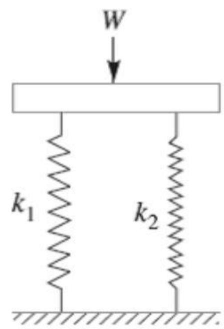
$$W = W_1 + W_2$$

$$\delta . k = \delta . k_1 + \delta . k_2$$

$$k = k_1 + k_2$$

$k$ = Combined stiffness of the springs, and

$\delta$ = Deflection produced.




**Surge in Springs or finding natural frequency of a helical spring:**

When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the beginning, the end coils of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end.

This wave of compression travels along the spring indefinitely. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engines and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and correspondingly very high stresses. Under these conditions, it is just possible that the spring may fail. This phenomenon is called ***surge*.**

It has been found that the natural frequency of spring should be at least twenty times the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies up to twentieth order. The natural frequency for springs clamped between two plates is given by

$$f_n = \frac{d}{2\pi D^2 . n} \sqrt{\frac{6\, G . g}{\rho}} \text{ cycles/s}$$

Where $d$ = Diameter of the wire,

$D$ = Mean diameter of the spring,

$n$ = Number of active turns,

$G$ = Modulus of rigidity,

$g$ = Acceleration due to gravity, and

$\rho$ = Density of the material of the spring.

The surge in springs may be eliminated by using the following methods:

**1.** By using friction dampers on the centre coils so that the wave propagation dies out.

**2.** By using springs of high natural frequency.

**3.** By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

**References:**


1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurmi

3. Design Data hand Book - S MD Jalaludin.

**Energy Stored in Helical Springs of Circular Wire**

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let $W$ = Load applied on the spring, and

$\delta$ = Deflection produced in the spring due to the load $W$.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8 W.D}{\pi d^3} \text{ or } W = \frac{\pi d^3.\tau}{8 K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8 W.D^3.n}{G.d^4} = \frac{8 \times \pi d^3.\tau}{8 K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi \tau.D^2.n}{K.d.G}$$

Substituting the values of $W$ and $\delta$ in equation ***(i)***, we have

$$U = \frac{1}{2} \times \frac{\pi d^3.\tau}{8 K.D} \times \frac{\pi \tau.D^2.n}{K.d.G}$$

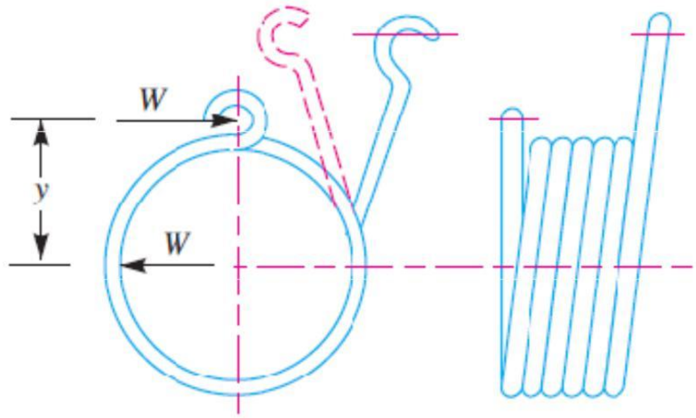
$$= \frac{\tau^2}{4 K^2.G} (\pi D.n) \left( \frac{\pi}{4} \times d^2 \right) = \frac{\tau^2}{4 K^2.G} \times V$$

Where $V$ = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

**Helical Torsion Springs**

The helical torsion springs as shown in Fig., may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion springs is bending stress whereas in compression or tension springs, the stresses are torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as in door hinges, brush holders in electric motors, automobile starters etc. A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is




$$\sigma_b = K \times \frac{32\ M}{\pi\ d^3} = K \times \frac{32\ W.y}{\pi\ d^3}$$

Where K = Wahl's stress factor = $\frac{4C^2\ \ C\ \ 1}{4C^{\ 2}\ \ \ 4C}$

C = Spring index,

M = Bending moment = W × y,

W = Load acting on the spring,

y = Distance of load from the spring axis, and

d = Diameter of spring wire.

And

Total angle of twist or angular deflection,

$$*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi\ d^4 / 64} = \frac{64\ M.D.n}{E.d^4}$$

Where l = Length of the wire = π.D.n,

—

D = Diameter of the spring, and

n = Number of turns.

And deflection,

$$\delta = \theta \times y = \frac{64\ M.D.n}{E.d^4} \times y$$

When the spring is made of rectangular wire having width b and thickness t, then

$$\sigma_b = K \times \frac{6\ M}{t.b^2} = K \times \frac{6\ W \times y}{t.b^2}$$

Where

$$K = \frac{3C^2 \quad C \quad 0.8}{3C^2 - 3C}$$

Angular deflection,

$$\theta = \frac{12\,\pi M.D.n}{E.t.b^3}; \quad \text{and} \quad \delta = \theta.y = \frac{12\,\pi M.D.n}{E.t.b^3} \times y$$

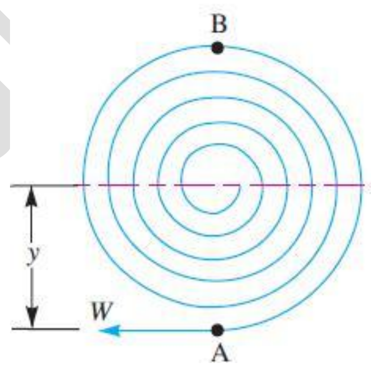
In case the spring is made of square wire with each side equal to b, then substituting t = b, in the above relation, we have

$$\sigma_b = K \times \frac{6\,M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12\,\pi M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12\,\pi M.D.n}{E.b^4} \times y$$

**<u>Flat Spiral Spring</u>**

A flat spring is a long thin strip of elastic material wound like a spiral as shown in Fig. These springs are frequently used in watches and gramophones etc. When the outer or inner end of this type of spring is wound up in such a way that there is a tendency in the increase of number of spirals of the spring, the strain energy is stored into its spirals. This energy is utilised in any useful way while the spirals open out slowly. Usually the inner end of spring is clamped to an arbor while the outer end may be pinned or clamped. Since the radius of curvature of every spiral decreases when the spring is wound up, therefore the material of the spring is in a state of pure bending. Let *W* = Force applied at the outer end *A* of the spring,




$y$ = Distance of centre of gravity of the spring from

$A$, $l$ = Length of strip forming the spring,

$b$ = Width of strip,

$t$ = Thickness of strip,

$I$ = Moment of inertia of the spring section = $b.t^3/12$,

and $Z$ = Section modulus of the spring section = $b.t^2/6$

When the end *A* of the spring is pulled up by a force *W*, then the bending moment on the spring, at a distance *y* from the line of action of *W* is given by

$$M = W \times y$$

The greatest bending moment occurs in the spring at *B* which is at a maximum distance from the application of *W*.

Bending moment at *B*,

$$M_B = M_{max} = W \times 2y = 2W.y = 2M$$

Maximum bending stress induced in the spring material,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{2W \times y}{b.t^2 / 6} = \frac{12W.y}{b.t^2} = \frac{12M}{b.t^2}$$

Assuming that both ends of the spring are clamped, the angular deflection (in radians) of the spring is given by

$$\theta = \frac{M.l}{E.I} = \frac{12\ M.l}{E.b.t^3}$$

And the deflection,

$$\delta = \theta \times y = \frac{M.l.y}{E.I}$$

$$= \frac{12\ M.l.y}{E.b.t^3} = \frac{12W.y^2.l}{E.b.t^3} = \frac{\sigma_b.y.l}{E.t}$$

The strain energy stored in the spring

$$= \frac{1}{2} M.\theta = \frac{1}{2} M \times \frac{M.l}{E.I} = \frac{1}{2} \times \frac{M^2.l}{E.I}$$

$$= \frac{1}{2} \times \frac{W^2.y^2.l}{E \times bt^3 / 12} = \frac{6\ W^2.y^2.l}{E.b.t^3}$$

$$= \frac{6\ W^2.y^2.l}{E.b.t^3} \times \frac{24bt}{24bt} = \frac{144\ W^2y^2}{Eb^2t^4} \times \frac{b\,t\,l}{24}$$

... (Multiplying the numerator and denominator by 24 *bt*)

$$= \frac{(\sigma_b)^2}{24\ E} \times b\,t\,l = \frac{(\sigma_b)^2}{24\ E} \times \text{Volume of the spring}$$

**References:**


1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurmi

3. Design Data hand Book - S MD Jalaludin.

Problem: A helical torsion spring of mean diameter 60 mm is made of a round wire of 6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stres s induced and the angular deflection of the spring in degrees. The spring index is 10 an d modulus of elasticity for the spring material is 200 kN/mm$^2$. The number of effective turns may be taken as 5.5.

**Solution.** Given : $D = 60$ mm ; $d = 6$ mm ; $M = 6$ N-m $= 6000$ N-mm ; $C = 10$ ; $E = 200$ kN/mm$^2$ $= 200 \times 10^3$ N/mm$^2$ ; $n = 5.5$

*Bending stress induced*

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

∴ Bending stress induced,

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = 1.08 \times \frac{32 \times 6000}{\pi \times 6^3} = 305.5 \text{ N/mm}^2 \text{ or MPa} \quad \textbf{Ans.}$$

*Angular deflection of the spring*

We know that the angular deflection of the spring (in radians),

$$\theta = \frac{64\,M.D.n}{E.d^4} = \frac{64 \times 6000 \times 60 \times 5.5}{200 \times 10^3 \times 6^4} = 0.49 \text{ rad}$$

$$= 0.49 \times \frac{180}{\pi} = 28° \quad \textbf{Ans.}$$

Problem: A spiral spring is mad e of a flat strip 6 mm wide and 0.25 mm thick. The length of the strip is 2.5 metres. Assumin g the maximum stress of 800 MPa to occur at the point of greatest bending moment, calculate the bending moment, the number of turns t o wind up the spring and the strain energy stor ed in the spring. Take E = 200 kN/mm$^2$.

*Bending moment in the spring*

Let $M$ = Bending moment in the spring.

We know that the maximum bending stress in the spring material ($\sigma_b$),

$$800 = \frac{12\,M}{b.t^2} = \frac{12\,M}{8\,(0.25)^2} = 32\,M$$

∴ $M = 800 / 32 = 25$ N-mm **Ans.**

*Number of turns to wind up the spring*

We know that the angular deflection of the spring,

$$\theta = \frac{12\,M.l}{E.b.t^3} = \frac{12 \times 25 \times 2500}{200 \times 10^3 \times 6\,(0.25)^3} = 40 \text{ rad}$$

Since one turn of the spring is equal to $2\pi$ radians, therefore number of turns to wind up the spring

$$= 40 / 2\pi = 6.36 \text{ turns } \textbf{Ans.}$$

*Strain energy stored in the spring*

We know that strain energy stored in the spring

$$= \frac{1}{2} M.\theta = \frac{1}{2} \times 24 \times 40 = 480 \text{ N-mm } \textbf{Ans.}$$

**Concentric or Composite Spri ngs or coaxial springs or nested springs**

A concentric or composite sprin g is used for one of the following purposes:

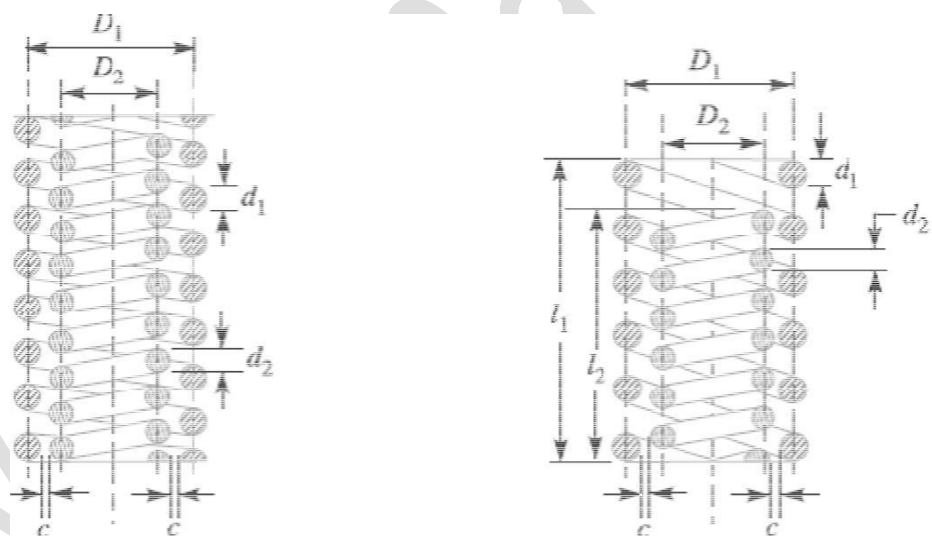
**1.** To obtain greater spring force within a given space.

**2.** To insure the operation of a mechanism in the event of failure of one of the springs.

The concentric springs for the above two purposes may have two or more spri ngs and have the same free lengths as shown i n Fig. (*a*) And are compressed equally.

Such springs are used in autom obile clutches; valve springs in aircraft, heavy duty diesel engines and rail-road car susp ension systems. Sometimes concentric spring s are used to obtain a spring force which d oes not increase in a direct relation to the d eflection but increases faster. Such springs are made of different lengths as shown in Fig. (*b*). The shorter spring begins to act only after the longer spring is compressed to a certain amount. These springs are used in governors of variable speed engines to take care of the variab le centrifugal force. The adjacent coils of t he concentric spring are wound in opposite directions to eliminate any tendency to bind.

If the same material is used, the concentric springs are designed for the same s tress. In order to get the same stress factor ($K$), it is desirable to have the same spring index ($C$ ).




(*a*) (*b*)

Consider a concentric spring as s hown in Fig. (*a*).

Let $W$ = Axial load,

$W_1$ = Load shared by outer sprin g,

$W_2$ = Load shared by inner sprin g,

$d_1$= Diameter of spring wire of outer spring,

$d_2$ = Diameter of spring wire of inner spring,

$D_1$ = Mean diameter of outer spring,

$D_2$ = Mean diameter of inner spring,

$\delta_1$ = Deflection of outer spring,

$\delta_2$ = Deflection of inner spring,

$n_1$ = Number of active turns of outer spring, and

$n_2$ = Number of active turns of in ner spring.

Assuming that both the springs are made of same material, then the maximu m shear stress induced in both the springs is ap proximately same, *i.e.*

$$\tau_1 = \tau_2$$

$$\frac{8\,W_1 . D_1 . K_1}{\pi\,(d_1)^3} = \frac{8\,W_2 . D_2 . K_2}{\pi\,(d_2)^3}$$

When stress factor, $K_1 = K_2$, then

$$\frac{W_1 . D_1}{(d_1)^3} = \frac{W_2 . D_2}{(d_2)^3}$$

If both the springs are effective throughout their working range, then their free length and deflection are equal, *i.e.*

$$\delta_1 = \delta_2$$

$$\frac{8 W_1\,(D_1)^3\,n_1}{(d_1)^4\,G} = \frac{8 W_2\,(D_2)^3\,n_2}{(d_2)^4\,G} \quad \text{or} \quad \frac{W_1\,(D_1)^3\,n_1}{(d_1)^4} = \frac{W_2\,(D_2)^3\,n_2}{(d_2)^4} \qquad ...(ii)$$

When both the springs are comp ressed until the adjacent coils meet, then the solid length of both the springs is equal, *i.e.*

$n_1.d_1 = n_2.d_2$

The equation ***(ii)*** may be written as

$$\frac{W_1\,(D_1)^3}{(d_1)^5} = \frac{W_2\,(D_2)^3}{(d_2)^5} \qquad ...(iii)$$

Now dividing equation ***(iii)*** by e quation ***(i)***, we have

$$\frac{(D_1)^2}{(d_1)^2} = \frac{(D_2)^2}{(d_2)^2} \quad \text{or} \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = C, \text{ the spring index} \qquad ...(iv)$$

*i.e.* the springs should be designed in such a way that the spring index for both the springs is same. From equations ***(i)*** and ***(iv)***, we have

$$\frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2} \quad \text{or} \quad \frac{W_1}{W_2} = \frac{(d_1)^2}{(d_2)^2} \qquad ..(v)$$

From Fig. 23.22 (*a*), we find tha the radial clearance between the two springs,

$$*c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually, the radial clearance bet ween the two springs is taken as

$$\frac{d_1 - d_2}{2}$$

$$\therefore \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

or $$\frac{D_1 - D_2}{2} = d_1 \qquad \text{------ (vi)}$$

From equation ***(iv)***, we find that

$$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

Substituting the values of $D_1$ and $D_2$ in equation ***(vi)***, we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\,d_1 = C.d_2$$

$$d_1\,(C-2) = C.d_2 \quad \text{or} \quad \frac{d_1}{d_2} = \frac{C}{C-2}$$

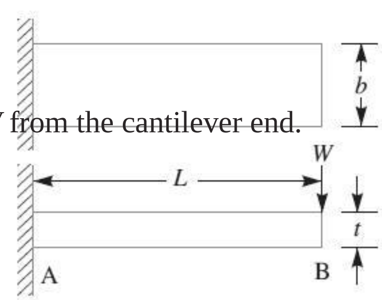
## Leaf Springs

Leaf springs (also known as **fla t springs**) are made out of flat plates. The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a d efinite path as it deflects to act as a structural member in addition to energy absorbing device. Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in additi on to shocks. Consider a single plate fixed at one end and loaded at the other end as show n in Fig. This plate may be used as a flat spring .

Let $t$ = Thickness of plate,

$b$ = Width of plate, and

$L$ = Length of plate or d istance of the load $W$ from the cantilever end.




We know that the maximum bending moment at the cantilever end $A$,

$$M = W.L$$

And section modulus,

$$Z = \frac{I}{y} = \frac{b\,t^3/12}{t/2} = \frac{1}{6} \times b.t^2$$

Bending stress in such a spring,

$$\sigma = \frac{M}{Z} = \frac{W.L}{\frac{1}{6} \times b.t^2} = \frac{6\,W.L}{b.t^2}$$

We know that the maximum deflection for a cantilever with concentrated load a t the free end is given by

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3/12} = \frac{4\,W.L^3}{E.b.t^3}$$

$$= \frac{2\,\sigma.L^2}{3\,E.t}$$

If the spring is not of cantilever type but it is like a simply supported beam, w ith length 2*L* and load 2*W* in the centre, as sh own in Fig. then Maximum bending moment in the centre,

*M* = *W.L*

Section modulus, *Z* = *b.t*2 / 6

Bending stress,

$$\sigma = \frac{M}{Z} = \frac{W.L}{b.t^2/6}$$

$$= \frac{6\,W.L}{b.t^2}$$

We know that maximum deflection of a simply supported beam loaded in the centre is given by

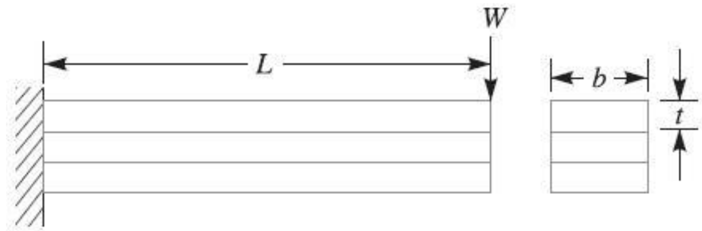
$$\delta = \frac{W_1\,(L_1)^3}{48\,E.I} = \frac{(2W)\,(2L)^3}{48\,E.I} = \frac{W.L^3}{3\,E.I}$$

From above we see that a spring such as automobile spring (semi-elliptical spring) with length 2*L* and loaded in the centre by a load 2*W*, may be treated as a double can tilever. If the plate of cantilever is cut into a series of *n* strips of width *b* and these are place d as shown in Fig., then equations ***(i)*** and ***(ii)*** m ay be written as

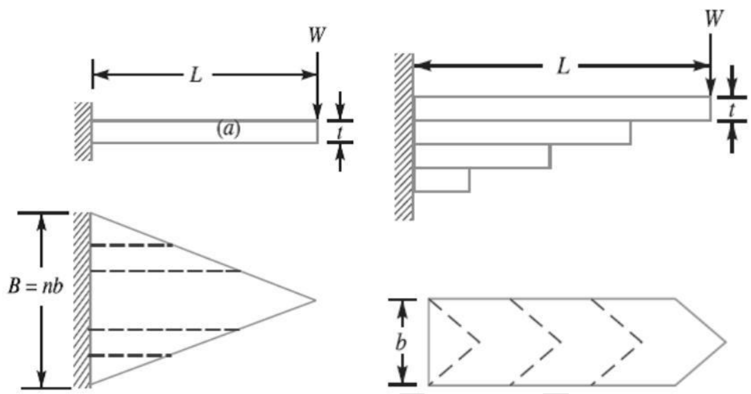
$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad ...(iii)$$

And

$$\delta = \frac{4\,W.L^3}{n.E.b.t^3} = \frac{2\,\sigma.L^2}{3\,E.t} \qquad ...(iv)$$




The above relations give the stress and deflection of a leaf spring of uniform cross section. The stress at such a spring is maximum at the support.




If a triangular plate is used as shown in Fig., the stress will be uniform throughout. If this triangular plate is cut into strips of uniform width and placed one below the other, as shown in Fig. to form a graduated or laminated leaf spring, then

$$\sigma = \frac{6\,W.L}{n.b.t^2} \qquad \ldots(v)$$

$$\delta = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \qquad \ldots(vi)$$

where $n$ = Number of graduated leaves.

A little consideration will show that by the above arrangement, the spring becomes compact so that the space occupied by the spring is considerably reduced.

When bending stress alone is considered, the graduated leaves may have zero width at the loaded end. But sufficient metal must be provided to support the shear. Therefore, it becomes necessary to have one or more leaves of uniform cross-section extending clear to the end. We see from equations ***(iv)*** and ***(vi)*** that for the same deflection, the stress in the uniform cross-section leaves (*i.e.* full length leaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects according to its own elastic curve. If the suffixes

F and G are used to indicate the full length (or uniform cross section) and gra duated leaves, then

$$\sigma_F = \frac{3}{2}\sigma_G$$

$$\frac{6 W_F . L}{n_F . b . t^2} = \frac{3}{2}\left[\frac{6\, W_G . L}{n_G . b . t^2}\right] \quad \text{or} \quad \frac{W_F}{n_F} = \frac{3}{2} \times \frac{W_G}{n_G}$$

$$\frac{W_F}{W_G} = \frac{3\, n_F}{2\, n_G} \qquad \text{...(vii)}$$

Adding 1 to both sides, we have

$$\frac{W_F}{W_G} + 1 = \frac{3\, n_F}{2\, n_G} + 1 \quad \text{or} \quad \frac{W_F + W_G}{W_G} = \frac{3\, n_F + 2\, n_G}{2\, n_G}$$

$$W_G = \left(\frac{2\, n_G}{3\, n_F + 2\, n_G}\right)(W_F + W_G) = \left(\frac{2\, n_G}{3\, n_F + 2\, n_G}\right) W \qquad \text{...(viii)}$$

where $W$ = Total load on the spring = $W_G + W_F$

$W_G$ = Load taken up by graduated leaves, and

$W_F$ = Load taken up by full length leaves.

From equation ***(vii)***, we may write

$$\frac{W_G}{W_F} = \frac{2\, n_G}{3\, n_F}$$

or

$$\frac{W_G}{W_F} + 1 = \frac{2\, n_G}{3\, n_F} + 1$$

$$\frac{W_G + W_F}{W_F} = \frac{2\, n_G + 3\, n_F}{3\, n_F}$$

$$\therefore \quad W_F = \left(\frac{3\, n_F}{2\, n_G + 3\, n_F}\right)(W_G + W_F) = \left(\frac{3\, n_F}{2\, n_G + 3\, n_F}\right) W \qquad \text{...(ix)}$$

Bending stress for full length leaves,

$$\sigma_F - \frac{6\, W_F . L}{n_F . b\, t^2} - \frac{6\, L}{n_F . b . t^2}\left(\frac{3\, n_F}{2\, n_G - 3\, n_F}\right) W - \frac{18\, W . L}{b . t^2\, (2\, n_G + 3\, n_F)}$$

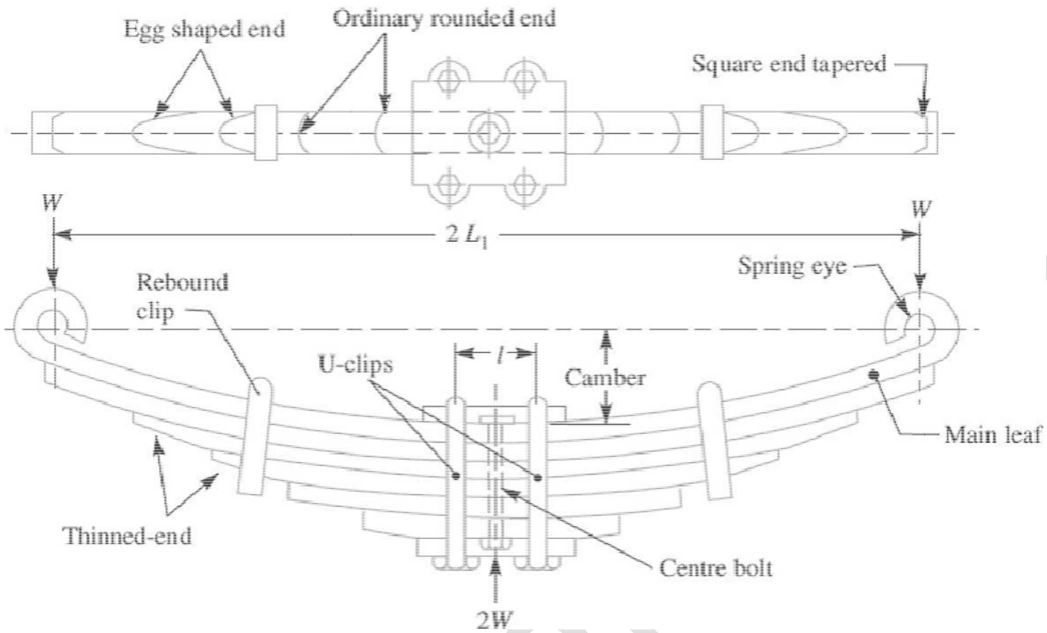
Since

$$\sigma_F - \frac{3}{2}\sigma_G, \text{ therefore}$$

$$\sigma_G = \frac{2}{3}\sigma_F = \frac{2}{3} \times \frac{18\, W . L}{b . t^2\, (2\, n_G + 3\, n_F)} = \frac{12\, W . L}{b . t^2\, (2\, n_G + 3\, n_F)}$$

The deflection in full length and graduated leaves is given by equation **(*iv*)**, *i.e*.

$$\delta = \frac{2\,\sigma_F \times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G + 3\,n_F)}$$



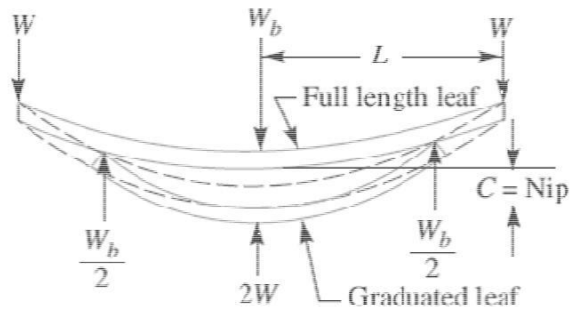

## Equalised Stress in Spring Leaves (Nipping)

We have already discussed that the stress in the full length leaves is 50% gr eater than the stress in the graduated leaves. In order to utilise the material to the best advantage, all the leaves should be equally stressed .

This condition may be obtained i n the following two ways:

**1.** By making the full length leaves of smaller thickness than the graduated leaves. In this way, the full length leaves will induce smaller bending stress due to small dist ance from the neutral axis to the edge of the lea f.

**2.** By giving a greater radius of curvature to the full length leaves than graduated leaves, as shown in Fig. before the leaves are assembled to form a spring. By doing so, a gap or clearance will be left between the leaves. This initial gap, as shown by *C* in Fig, is called ***nip*.**




Consider that under maximum load conditions, the stress in all the leaves is equal. Then at maximum load, the total deflection of the graduated leaves will exceed the deflection of the full length leaves by an amount qual to the initial gap *C*. In other words,

$$\delta_G = \delta_F + C$$

$$C = \delta_G - \delta_F = \frac{6\,W_G\,.\,L^3}{n_G\,E.b.t^3} - \frac{4\,W_F.L^3}{n_F.E.b.t^3} \qquad ...(i)$$

Since the stresses are equal, ther efore

$$\sigma_G = \sigma_F$$

$$\frac{6\,W_G.L}{n_G\,b.t^2} = \frac{6\,W_F\,L}{n_F\,b.t^2} \quad \text{or} \quad \frac{W_G}{n_G} - \frac{W_F}{n_F}$$

$$\therefore \qquad W_G = \frac{n_G}{n_F} \times W_F = \frac{n_G}{n} \times W$$

$$W_F = \frac{n_F}{n_G} \times W_G = \frac{n_F}{n} \times W$$

Substituting the values of *W*G an d *W*F in equation ***(i)***, we have

$$C = \frac{6W.L^3}{n.E.b.t^3} - \frac{4W.L^3}{n.E.b.t^3} = \frac{2W.L^3}{n.E.b.t^3} \qquad ...(ii)$$

The load on the clip bolts ($W_b$) r equired to close the gap is determined by the fact that the gap is equal to the initial deflections of full length and graduated leaves.

$$\therefore \qquad C = \delta_F + \delta_G$$

$$\frac{2W.L^3}{n.E.b.t^3} = \frac{4L^3}{n_F.E.b.t^3} \times \frac{W_b}{2} + \frac{6\,L^3}{n_G.E.b.t^3} \times \frac{W_b}{2}$$

Or

$$\frac{W}{n} = \frac{W_b}{n_F} + \frac{3\,W_b}{2\,n_G} = \frac{2\,n_G.W_b + 3\,n_F.W_b}{2\,n_F.n_G} = \frac{W_b\,(2\,n_G + 3\,n_F)}{2\,n_F.n_G}$$

$$W_b = \frac{2\,n_F.n_G.W}{n\,(2\,n_G + 3\,n_F)} \qquad \text{...}(iii)$$

The final stress in spring leaves will be the stress in the full length leaves due to the applied load ***minus*** the initial stress.

Final stress,

$$\sigma = \frac{6\,W_F.L}{n_F.b.t^2} - \frac{6\,L}{n_F.b.t^2} \times \frac{W_b}{2} = \frac{6\,L}{n_F.b.t^2}\left(W_F - \frac{W_b}{2}\right)$$

$$= \frac{6\,L}{n_F.b.t^2}\left[\frac{3n_F}{2n_G + 3n_F} \times W - \frac{n_F.n_G.W}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6W.L}{b.t^2}\left[\frac{3}{2n_G + 3n_F} - \frac{n_G}{n\,(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3n - n_G}{n(2n_G + 3n_F)}\right]$$

$$= \frac{6\,W.L}{b.t^2}\left[\frac{3(n_F + n_G) - n_G}{n(2n_G + 3n_F)}\right] = \frac{6\,W.L}{n.b.t^2} \qquad \text{...}(iv)$$

**Length of Leaf Spring Leaves**

The length of the leaf spring leav es may be obtained as discussed below :

Let $2L_1$ = Length of span or overall length of the spring,

$l$ = Width of band or dist ance between centres of *U*-bolts. It is the in effective length of the spring,

$n_F$ = Number of full leng th leaves,

$n_G$ = Number of graduated leaves, and

$n$ = Total number of leaves = $n_F + n_G$.

We have already discuss ed that the effective length of the spring,

$2L = 2L_1 - l$ ... (When band is used)