Design of Machine Elements

Lecture Notes

Unit-1

Introduction

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one. From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings. In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

Classifications of Machine Design

The machine design may be classified as follows:

1. ***Adaptive design.*** In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

2. ***Development design.*** This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. ***New design.*** This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design. The designs, depending upon the methods used, may be classified as follows:

(*a*) ***Rational design.*** This type of design depends upon mathematical formulae of principle of mechanics.

(*b*) ***Empirical design.*** This type of design depends upon empirical formulae based on the practice and past experience.

(*c*) ***Industrial design.*** This type of design depends upon the production aspects to manufacture any machine component in the industry.

(*d*) *Optimum design.* It is the best design for the given objective function under the specified constraints. It may be achieved by minimising the undesirable effects.

(*e*) *System design.* It is the design of any complex mechanical system like a motor car.

(*f*) *Element design.* It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

(*g*) *Computer aided design.* This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimisation of a design.

General Considerations in Machine Design

Following are the general considerations in designing a machine component:

1. ***Type of load and stresses caused by the load.*** The load, on a machine component, may act in several ways due to which the internal stresses are set up. The various types of load and stresses are discussed later.

2. ***Motion of the parts or kinematics of the machine.*** The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required.

The motion of the parts may be:

(*a*) Rectilinear motion which includes unidirectional and reciprocating motions.

(*b*) Curvilinear motion which includes rotary, oscillatory and simple harmonic. (*c*) Constant velocity.

(*d*) Constant or variable acceleration.

3. ***Selection of materials.*** It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are: strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc. The various types of engineering materials and their properties are discussed later.

4. ***Form and size of the parts.*** The form and size are based on judgment. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

5. ***Frictional resistance and lubrication.*** There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

6. ***Convenient and economical features.*** In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible. The economical operation of a machine which is to be used for production or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. ***Use of standard parts.*** The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

8. ***Safety of operation.*** Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

9. ***Workshop facilities.*** A design engineer should be familiar with the limitations of this employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

10. ***Number of machines to be manufactured.*** The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

11. ***Cost of construction.*** The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of handmade samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions should be to reduce the manufacturing cost to the minimum.

12. ***Assembling.*** Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows:

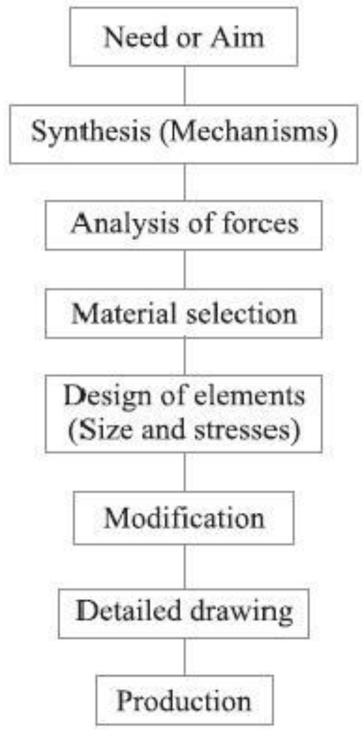

Fig.1. General Machine Design Procedure

1. *Recognition of need.* First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. *Synthesis* (*Mechanisms*). Select the possible mechanism or group of mechanisms which will give the desired motion.

3. *Analysis of forces.* Find the forces acting on each member of the machine and the energy transmitted by each member.

4. *Material selection.* Select the material best suited for each member of the ma chine.

5. *Design of elements* (*Size and Stresses*). Find the size of each member of the machine by considering the force acting on t he member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than th e permissible limit.

6. *Modification.* Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. *Detailed drawing.* Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. *Production.* The component, as per the drawing, is manufactured in the workshop.
The flow chart for the general procedure in machine design is shown in Fig.

Note: When there are number of components in the market having the same qualities of efficiency, durability and cost, then the customer will naturally attract towards the most appealing product. The aesthetic and ergonomics are very important features which gives grace and lustre to product and dominates the market.

Engineering materials and their properties

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. Now, we shall discuss the commonly used engineering materials and their properties in Machine Design.

Classification of Engineering Materials

The engineering materials are mainly classified as:

1. Metals and their alloys, such as iron, steel, copper, aluminum, etc.

2. Non-metals, such as glass, rubber, plastic, etc.

The metals may be further classified as:

(a) Ferrous metals and (*b*) Non-ferrous metals.

The ******ferrous metals*** are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The ***non-ferrous*** metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.

Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:

1. Availability of the materials,

2. Suitability of the materials for the working conditions in service, and

3. The cost of the materials.

The important properties, which determine the utility of the material, are physical, chemical and mechanical properties. We shall now discuss the physical and mechanical properties of the material in the following articles.

Physical Properties of Metals

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

1. Strength. It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

2. Stiffness. It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

3. Elasticity. It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. Plasticity. It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility. It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. Brittleness. It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

7. Malleability. It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. Toughness. It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable in parts subjected to shock and impact loads.

9. Machinability. It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material. It may be noted that brass can be easily machined than steel.

10. Resilience. It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. Creep. When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called **creep.** This property is considered in designing internal combustion engines, boilers and turbines.

12. Fatigue. When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as ***fatigue**. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

13. Hardness. It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

(a) Brinell hardness test,

(b) Rockwell hardness test,

(c) Vickers hardness (also called Diamond Pyramid) test, and

(d) Shore scleroscope.

Steel

It is an alloy of iron and carbon, with carbon content up to a maximum of 1.5%. The carbon occurs in the form of iron carbide, because of its ability to increase the hardness and strength of the steel. Other elements *e.g.* silicon, sulphur, phosphorus and manganese are also present to greater or lesser amount to impart certain desired properties to it. Most of the steel produced now-a-days is ***plain carbon steel*** or simply ***carbon steel*.** A carbon steel is defined as a steel which has its properties mainly due to its carbon content and does not contain more than 0.5% of silicon and 1.5% of manganese.

The plain carbon steels varying from 0.06% carbon to 1.5% carbon are divided into the following types depending upon the carbon content.

1. Dead mild steel — up to 0.15% carbon

2. Low carbon or mild steel — 0.15% to 0.45% carbon

3. Medium carbon steel — 0.45% to 0.8% carbon

4. High carbon steel — 0.8% to 1.5% carbon

According to Indian standard *[IS: 1762 (Part-I)–1974], a new system of designating the steel is recommended. According to this standard, steels are designated on the following two basis: (*a*) On the basis of mechanical properties, and (*b*) On the basis of chemical composition. We shall now discuss, in detail, the designation of steel on the above two basis, in the following pages.

Steels Designated on the Basis of Mechanical Properties

These steels are carbon and low alloy steels where the main criterion in the selection and inspection of steel is the tensile strength or yield stress. According to Indian standard IS: 1570 (Part–I)- 1978 (Reaffirmed 1993), these steels are designated by a symbol 'Fe' or 'Fe E' depending on whether the steel has been specified on the basis of minimum tensile strength or yield strength, followed by the figure indicating the minimum tensile strength or yield stress in N/mm2. For example 'Fe 290' means a steel having minimum tensile strength of 290 N/mm2 and 'Fe E 220' means a steel having yield strength of 220 N/mm2.

Steels Designated on the Basis of Chemical Composition

According to Indian standard, IS : 1570 (Part II/Sec I)-1979 (Reaffirmed 1991), the carbon steels are designated in the following order :

(*a*) Figure indicating 100 times the average percentage of carbon content,

(*b*) Letter 'C', and

(*c*) Figure indicating 10 times the average percentage of manganese content. The figure after multiplying shall be rounded off to the nearest integer.

For example 20C8 means a carbon steel containing 0.15 to 0.25 per cent (0.2 per cent on average) carbon and 0.60 to 0.90 per cent (0.75 per cent rounded off to 0.8 per cent on an average) manganese.

Effect of Impurities on Steel

The following are the effects of impurities like silicon, sulphur, manganese and phosphorus on steel.

1. ***Silicon*****.** The amount of silicon in the finished steel usually ranges from 0.05 to 0.30%. Silicon is added in low carbon steels to prevent them from becoming porous. It removes the gases and oxides, prevent blow holes and thereby makes the steel tougher and harder.

2. ***Sulphur*****.** It occurs in steel either as iron sulphide or manganese sulphide. Iron sulphide because of its low melting point produces red shortness, whereas manganese sulphide does not affect so much. Therefore, manganese sulphide is less objectionable in steel than iron sulphide.

3. ***Manganese*****.** It serves as a valuable deoxidising and purifying agent in steel. Manganese also combines with sulphur and thereby decreases the harmful effect of this element remaining in the steel. When used in ordinary low carbon steels, manganese makes the metal ductile and of good bending qualities. In high speed steels, it is used to toughen the metal and to increase its critical temperature.

4. ***Phosphorus*****.** It makes the steel brittle. It also produces cold shortness in steel. In low carbon steels, it raises the yield point and improves the resistance to atmospheric corrosion. The sum of carbon and phosphorus usually does not exceed 0.25%.

References:

1. Machine Design - V.Bandari .
2. Machine Design – R.S. Khurmi
3. Design Data hand Book - S MD Jalaludin.

Manufacturing considerations in Machine design Manufacturing Processes

The knowledge of manufacturing processes is of great importance for a design engineer. The following are the various manufacturing processes used in Mechanical Engineering.

1. ***Primary shaping processes*****.** The processes used for the preliminary shaping of the machine component are known as primary shaping processes. The common operations used for this process are casting, forging, extruding, rolling, drawing, bending, shearing, spinning, powder metal forming, squeezing, etc.

2. ***Machining processes*****.** The processes used for giving final shape to the machine component, according to planned dimensions are known as machining processes. The common operations used for this process are turning, planning, shaping, drilling, boring, reaming, sawing, broaching, milling, grinding, hobbing, etc.

3. ***Surface finishing processes*****.** The processes used to provide a good surface finish for the machine component are known as surface finishing processes. The common operations used for this process are polishing, buffing, honing, lapping, abrasive belt grinding, barrel tumbling, electroplating, super finishing, sheradizing, etc.

4. ***Joining processes*****.** The processes used for joining machine components are known as joining processes. The common operations used for this process are welding, riveting, soldering, brazing, screw fastening, pressing, sintering, etc.

5. ***Processes effecting change in properties*****.** These processes are used to impart certain specific properties to the machine components so as to make them suitable for particular operations or uses. Such processes are heat treatment, hot-working, cold-working and shot peening.

Other considerations in Machine design

1. Workshop facilities.
2. Number of machines to be manufactured
3. Cost of construction

4. Assembling

Interchangeability

The term interchangeability is normally employed for the mass productio n of identical items within the prescribed limi ts of sizes. A little consideration will show that in order to maintain the sizes of the part within a close degree of accuracy, a lot of time is required. But even then there will be small va riations. If the variations are within certain limits, all parts of equivalent size will be equally fit for operating in machines and mechanism s. Therefore, certain variations are recognized and allowed in the sizes of the mating part s to give the required fitting. This facilitates to select at random from a large number of parts for an assembly and results in a consid erable saving in the cost of production.

In order to control the size of finished part, with due allowance for error, for interchangeable parts is called ***limit system.*** It may be noted that when an assembly is made of two parts, the part which enters into the other, is known as ***enveloped surfac e*** (or **shaft** for cylindrical part) and the other i n which one enters is called ***enveloping surface*** (or **hole** for cylindrical part). The term ***shaft*** refers not only to the diameter of a circular s haft, but it is also used to designate any external dimension of a part. The term ***hole*** refers not only to the diameter of a circular hole, but it is also used to designate any internal dimensio n of a part.

Important Terms used in Limit System

The following terms used in limit system (or interchangeable system) are important from the subject point of view:

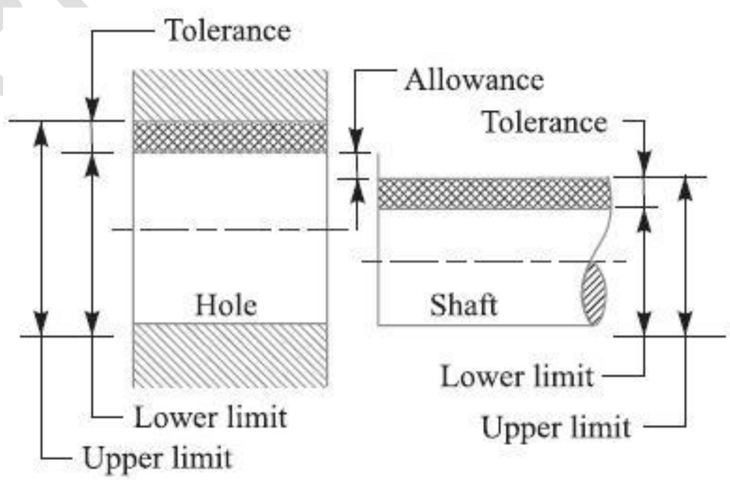

Fig. Limits of sizes.

1. Nominal size. It is the size of a part specified in the drawing as a matter of convenience.

2. Basic size. It is the size of a part to which all limits of variation (i.e. tolerances) are applied to arrive at final dimensioning of the mating parts. The nominal or basic size of a part is often the same.

3. Actual size. It is the actual measured dimension of the part. The differencee between the basic size and the actual size sho uld not exceed a certain limit; otherwise it will interfere with the interchangeability of the mating parts.

4. Limits of sizes. There are tw o extreme permissible sizes for a dimension of the part as shown in Fig. The largest permi sible size for a dimension of the part is called **u pper** or **high** or **maximum limit,** whereas the smallest size of the part is known as **lower** or **minimum limit.**

5. Allowance. It is the differen ce between the basic dimensions of the mati ng parts. The allowance may be **positive** or **negative**. When the shaft size is less than the hole size, then the allowance is **positive** and when the shaft size is greater than the hole size, then the allowance is **negative.**

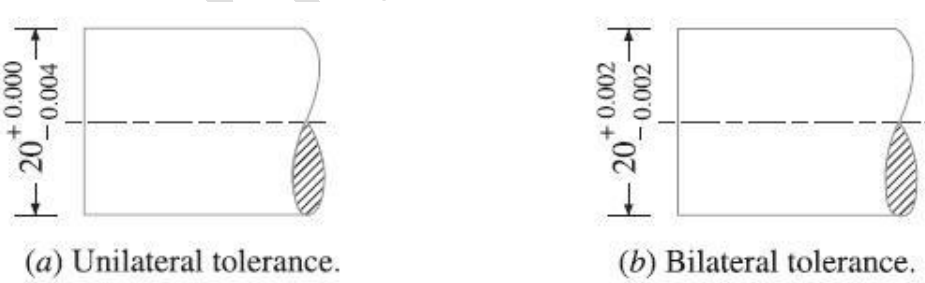

Fig. Method of assigning Tolerances

6. Tolerance. It is the differenc e between the upper limit and lower limit of a dimension. In other words, it is the maximum permissible variation in a dimension. The tolerance may be **unilateral** or **bilateral.** When all the tolerance is allowed on one side of the nom inal size, e.g. $20^{0.000}_{0.004}$, then it is said to be **unilateral system of tolerance.** The unilateral system is mostly used in industries as it permits changing the tolerance value while still retain ing the same

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allowance or type of fit. When t he tolerance is allowed on both sides of the nom inal size, e.g. $20^{0.002}_{0.002}$, then it is said to be **bil ateral system of tolerance.** In this case + 0.00 2 is the upper limit and – 0.002 is the lower lim it.

7. Tolerance zone. It is the zone between the maximum and minimum limit size .

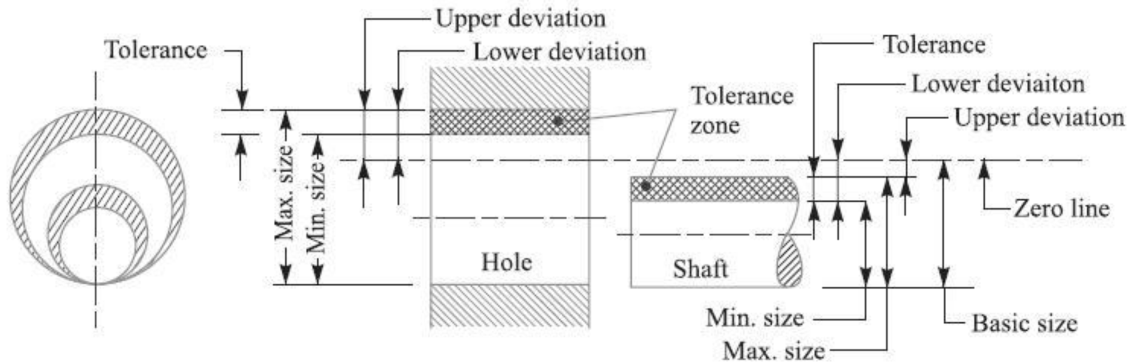


Fig. Tolerance Zone

8. Zero line. It is a straight line corresponding to the basic size. The deviations are measured from this line. The positive and negative deviations are shown above and below the zero line respectively.

9. Upper deviation. It is the algebraic difference between the maximum size and the basic size. The upper deviation of a hole is represented by a symbol ES (Ecart Superior) and of a shaft, it is represented by es.

10. Lower deviation. It is the algebraic difference between the minimum size and the basic size. The lower deviation of a h ole is represented by a symbol EI (Ecart Inferior) and of a shaft, it is represented by ei.

11. Actual deviation. It is the algebraic difference between an actual size and the corresponding basic size.

12. Mean deviation. It is the arithmetical mean between the upper and lower deviations.

13. Fundamental deviation. It is one of the two deviations which are conventio nally chosen to define the position of the tolerance zone in relation to zero line, as shown in Fig.

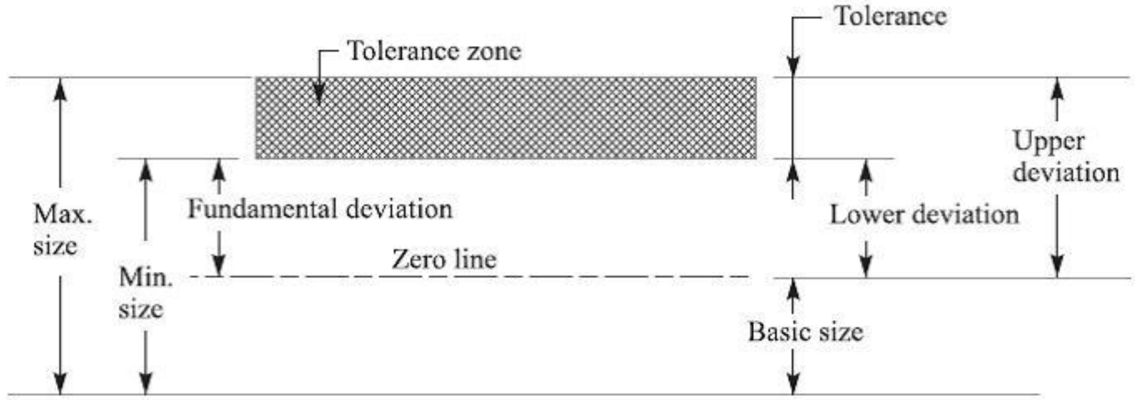
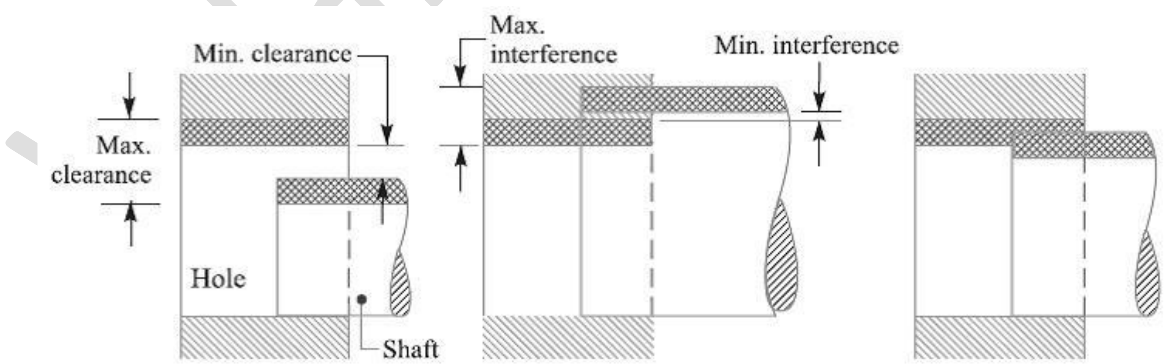


Fig. Fundamental deviation.

Fits

The degree of tightness or loos eness between the two mating parts is known as a *fit* of the parts. The nature of fit is characterized by the presence and size of clearance and interference. The ***clearance*** is the amount by which the actual size of the shaft is less than the actual size of the mating hole in an assemb ly as shown in Fig. 3.5 (*a*). In other words, the clearance is the difference between the size s of the hole and the shaft before assembly. T he difference must be ***positive*.**

The ***clearance*** is the amount by which the actual size of the shaft is less than the actual size of the mating hole in an assemb ly as shown in Fig. (*a*). In other words, the cl earance is the difference between the sizes of the hole and the shaft before assembly. The difference must be ***positive*.**



(*a*) Clearance fit. (*b*) Interference fit. (*c*) Transition fit.

Fig. Types of fits.

The ***interference*** is the amount by which the actual size of a shaft is larger th an the actual finished size of the mating hol e in an assembly as shown in Fig. (*b*). In other words, the interference is the arithmetical difference between the sizes of the hole and the shaft, before assembly. The difference must be ***negative*.**

Types of Fits

According to Indian standards, the fits are classified into the following three groups:

1. Clearance fit*.** In this type of fit, the size limits for mating parts are so selected that clearance between them always occur, as shown in Fig. (*a*). It may be noted that in a clearance fit, the tolerance zone of the hole is entirely above the tolerance zone of the shaft. In a clearance fit, the difference between the minimum size of the hole and the maximum size of the shaft is known as ***minimum clearance whereas the difference between the maximum size of the hole and minimum size of the shaft is called ***maximum clearance*** as shown in Fig. (*a*). The clearance fits may be slide fit, easy sliding fit, running fit, slack running fit and loose running fit.

2. ***Interference fit*.** In this type of fit, the size limits for the mating parts are so selected that interference between them always occur, as shown in Fig. (*b*). It may be noted that in an interference fit, the tolerance zone of the hole is entirely below the tolerance zone of the shaft. In an interference fit, the difference between the maximum size of the hole and the minimum size of the shaft is known as ***minimum interference*,** whereas the difference between the minimum size of the hole and the maximum size of the shaft is called ***maximum interference*,** as shown in Fig. (*b*).

The interference fits may be shrink fit, heavy drive fit and light drive fit.

3. ***Transition fit*.** In this type of fit, the size limits for the mating parts are so selected that either a clearance or interference may occur depending upon the actual size of the mating parts, as shown in Fig. (*c*). It may be noted that in a transition fit, the tolerance zones of hole and shaft overlap. The transition fits may be force fit, tight fit and push fit.

Basis of Limit System

The following are two bases of limit system:

1. ***Hole basis system*.** When the hole is kept as a constant member (*i.e*. when the lower deviation of the hole is zero) and different fits are obtained by varying the shaft size, as shown in Fig. (*a*), then the limit system is said to be on a hole basis.

2. ***Shaft basis system.*** When th e shaft is kept as a constant member (*i.e.* wh en the upper deviation of the shaft is zero) and different fits are obtained by varying the hole size, as shown in Fig.(*b*), Then the limit system is said to be on a shaft basis.

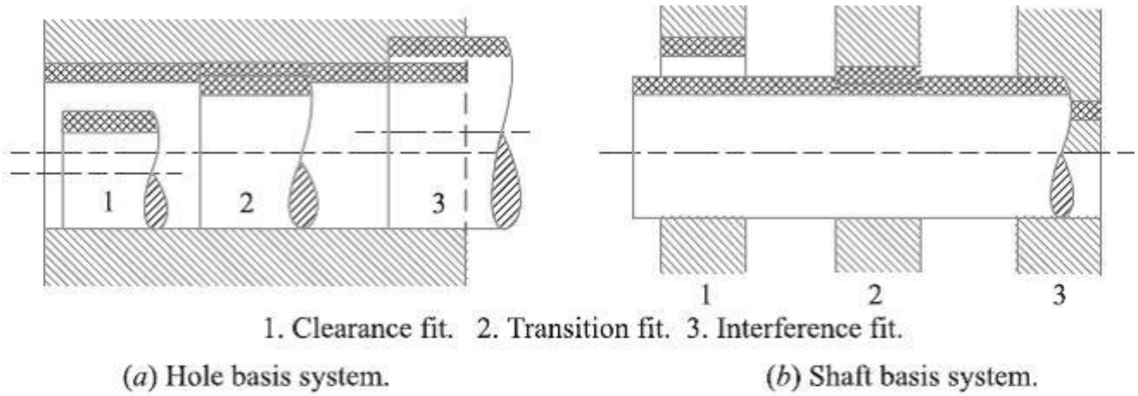


Fig. Bases of Limit System.

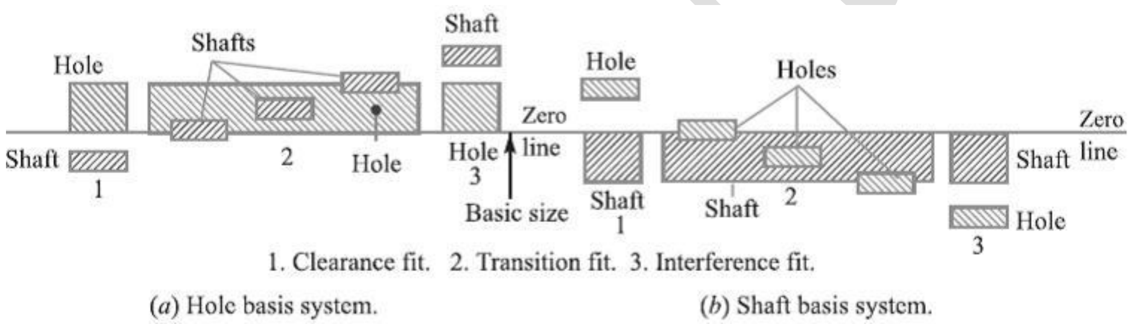


Fig. Bases of Limit System

The hole basis and shaft basis system may also be shown as in Fig. with respect to the zero line. It may be noted that from th e manufacturing point of view, a hole basis system is always preferred. This is because the holes are usually produced and finished by standar d tooling like drill, reamers, etc., whose size is not adjustable easily. On the other hand, the si ze of the shaft (which is to go into the hole) c an be easily adjusted and is obtained by turning or grinding operations.

References:

1. Machine Design - V.Bandari .
2. Machine Design – R.S. Khurm i
3. Design Data hand Book - S MD Jalaludin.

Stress Concentration:

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. His irregularity in the stress distribution caused by abrupt changes of form is called ***stress concentration.*** It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc. In order to understand fully the idea of stress concentration, consider a member with different cross-section under a t ensile load as shown in Fig. A little consideration will show that the nominal stress in the right and left hand sides will be uniform but in the region where the cross-section is changing, a re-distribution of the force within the member must take place. The material near the edges is stressed considerably higher than the average value. The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

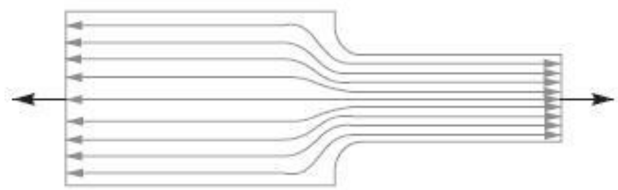


Fig. Stress concentration

Theoretical or Form Stress Concentration Factor

The theoretical or form stress concentration factor is defined as the ratio of t he maximum stress in a member (at a notch or a fillet) to the nominal stress at the same sectio n based upon net area. Mathematically, theoret ical or form stress concentration factor,

$$K_t = \text{Maximum stress/ Nominal stress}$$

The value of K_t depends upon the material and geometry of the part. In static loading, stress concentration in ductile materials is not so serious as in brittle materia ls, because in ductile materials local deformat ion or yielding takes place which reduces the concentration. In brittle materials, cracks may appear at these local concentrations of stresss which will increase the stress over the rest of the section. It is, therefore, necessary that in designing parts of brittle materials such as castings, care should be taken. In order to avoid failure due to stress concentration, fillets at the changes of section must be provided.

In cyclic loading, stress concentration in ductile materials is always serious because the ductility of the material is n ot effective in relieving the concentration of stress caused by cracks, flaws, surface roughnes s, or any sharp discontinuity in the geometrical form of the member. If the stress at any poin t in a member is above the endurance limit of the material, a crack may develop under the action of repeated load and the crack will lead to failure of the member.

Stress Concentration due to Holes and Notches

Consider a plate with transverse elliptical hole and subjected to a tensile load as shown in Fig.1(a). We see from the stress -distribution that the stress at the point away fro m the hole is practically uniform and the maximum stress will be induced at the edge of the hole. The maximum stress is given by

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{b}\right)$$

And the theoretical stress concentration factor,

$$K_t = \frac{\sigma_{max}}{\sigma} = \left(1 + \frac{2a}{r}\right)$$

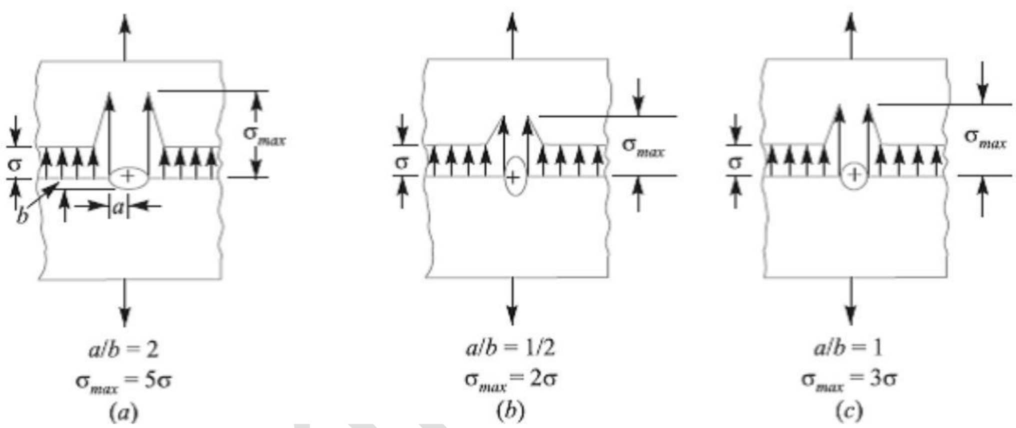


Fig.1. Stress concentration due to holes.

The stress concentration in the notched tension member, as shown in Fig. 2, is influenced by the depth a of the notch and radius r at the bottom of the notch. The maximum stress, which applies to members having notches that are small in comparison with the widt h of the plate, may be obtained by the following equation,

$$\sigma_{max} = \sigma\left(1 + \frac{2a}{r}\right)$$

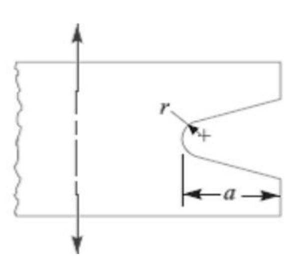


Fig.2. Stress concentration due to notches.

Methods of Reducing Stress Concentration

Whenever there is a change in cross-section, such as shoulders, holes, notches or keyways and where there is an interference fit between a hub or bearing race and a shaft, then stress concentration results. The presence of stress concentration can not be totally eliminated but it may be reduced to some extent. A device or concept that is useful in assisting a design engineer to visualize the presence of stress concentration and how it may be mitigated is that of stress flow lines, as shown in Fig.3. The mitigation of stress concentration means that the stress flow lines shall maintain their spacing as far as possible.

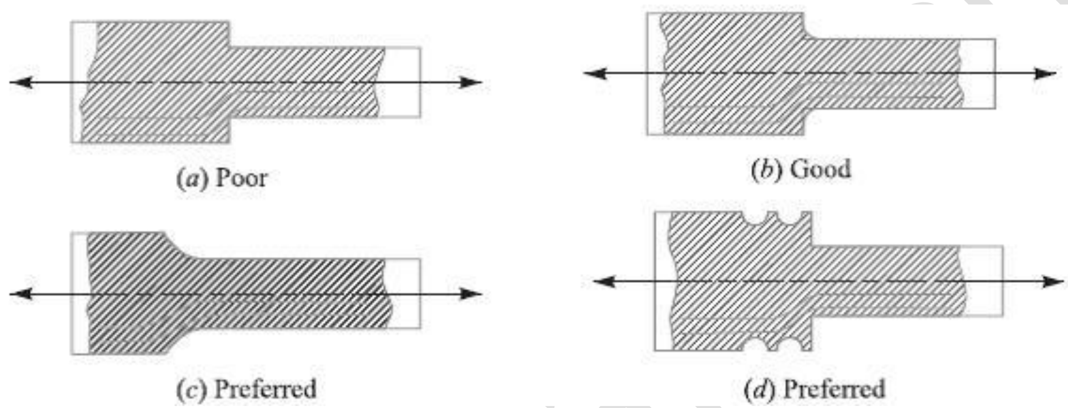


Fig.3

In Fig. 3 (a) we see that stress lines tend to bunch up and cut very close to the sharp re-entrant corner. In order to improve the situation, fillets may be provided, as shown in Fig. 3 (b) and (c) to give more equally spaced flow lines.

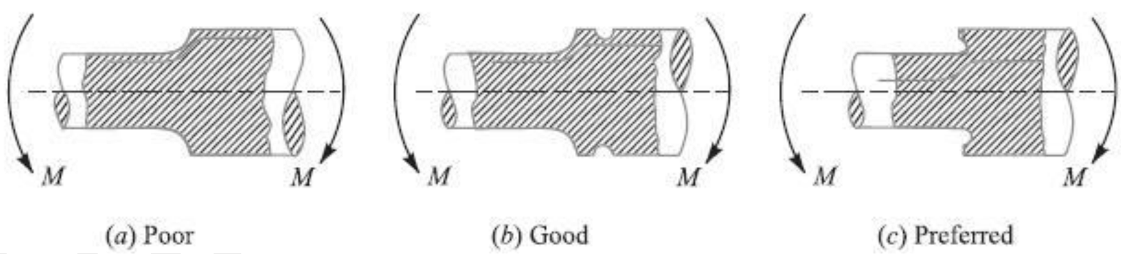


Fig. reducing stress concentration in cylindrical members with shoulders

Fig. Reducing stress concentration in cylindrical members with holes.

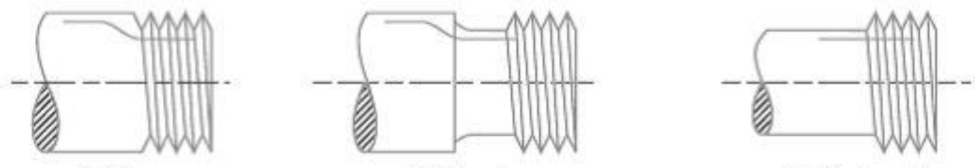


Fig. Reducing stress concentration in cylindrical members with hole s

Completely Reversed or Cyclic Stresses

Consider a rotating beam of circ ular cross-section and carrying a load *W*, as s hown in Fig1. This load induces stresses in the beam which are cyclic in nature. *A* little consideration will show that the upper fibres of th e beam (*i.e*. at point *A*) are under compressive stress and the lower fibres (*i.e*. at point *B*) ar e under tensile stress. After half a revolution, the point *B* occupies the position of point *A* and the point *A* occupies the position of point *B*. Thus the point *B* is now under compressive stress and the point *A* under tensile stress. The speed of variation of these stresses depen ds upon the speed of the beam.

From above we see that for each revolution of the beam, the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile or ***vice versa,*** are known as ***completely reversed*** or ***cyclic stresses.*** The stresses which vary from a min imum value to a maximum value of the same nature, (*i.e*. tensile or compressive) are called ***fluctuating stresses.*** The stresses which vary from zero to a certain maximum value are called ***repeated stresses.*** The stresses which vary from a minimum value to a maximum value of the opposite nature (*i.e*. from a certain minimum compressive to a certain maximum tensile or from a minimum tensile to a maximum compressive) are called ***alternati ng stresses.***

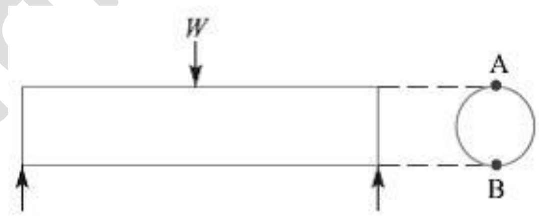


Fig.1. Shaft subjected to cyclic load

Fatigue and Endurance Limit

It has been found expe rimentally that when a material is subjectedd to repeated stresses; it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue.** The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. The failure may occur even without any pr ior indication. The fatigue of material is effect ed by the size of the component, relative magnitude of static and fluctuating loads and the nu mber of load reversals.

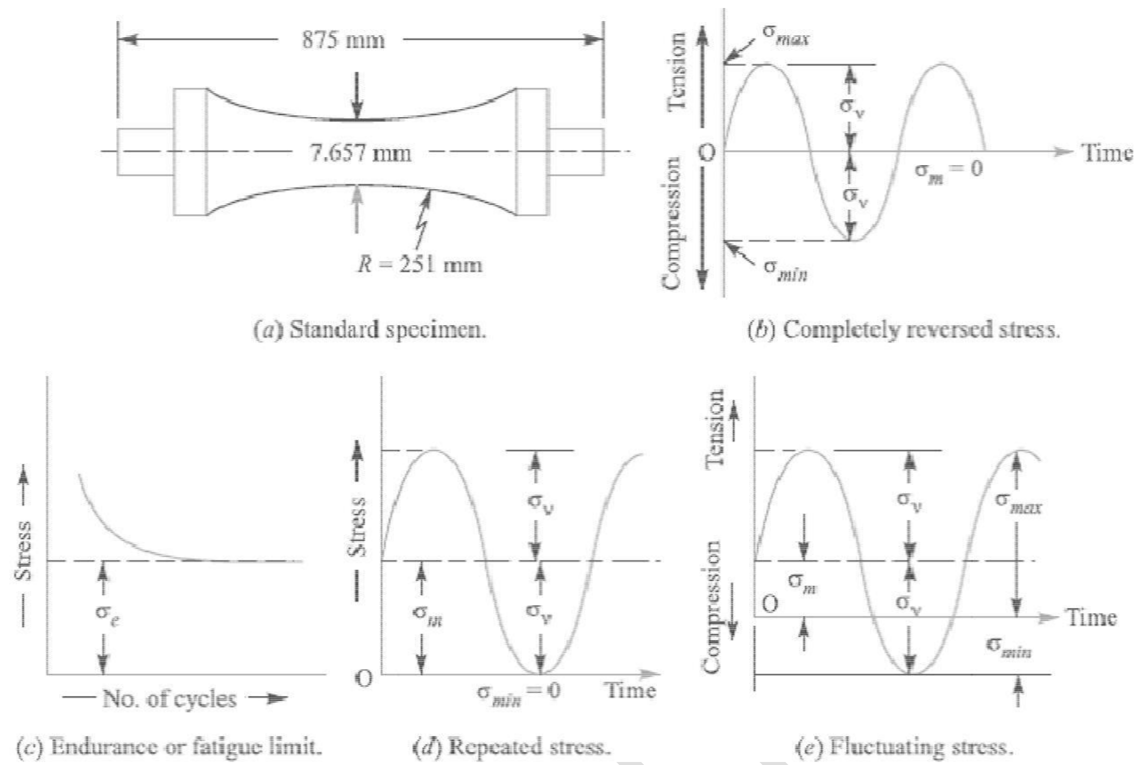


Fig.2. Time-stress diagrams.

In order to study the effe ct of fatigue of a material, a rotating mirror beam method is used. In this method, a standard mirror polished specimen, as shown in Fig.2 (*a*), is rotated in a fatigue testing machine while the specimen is loaded in bending. As the spe cimen rotates, the bending stress at the upper fi bres varies from maximum compressive to ma ximum tensile while the bending stress at th e lower fibres varies from maximum tensile to maximum compressive. In other words, the specimen is subjected to a completely reversed stress cycle. This is represented by a time-stress diagram as shown in Fig.2 (*b*). A record is kept of the number of cycles required to produce failure at a given stress, and the results are plotted in stress-cycle curve as shown in F ig.2 (*c*). A little consideration will show that if the stress is kept below a certain value as shown by dotted line in Fig.2 (*c*), the material will not fail whatever may be the number of cycles. This stress, as represented by dotted line , is known as ***endurance*** or ***fatigue limit*** (σe) It is defined as maximum value of the compl etely reversed bending stress which a polished standard specimen can withstand without failur e, for infinite number of cycles (usually 107 cy cles).

It may be noted that the term endurance limit is used for reversed bending only while for other types of loading, the term ***endurance strength*** may be used when referring the fatigue strength of the material. It may be defined as the safe maximum stress which can be applied to the machine part working under actual conditions.

We have seen that when a machine member is subjected to a completely reversed stress, the maximum stress in tension is equal to the maximum stress in compression as shown in Fig.2 (*b*). In actual practice, many machine members undergo different range of stress than the completely reversed stress. The stress ***verses*** time diagram for fluctuating stress having values σ_{min} and σ_{max} is shown in Fig.2 (*e*). The variable stress, in general, may be considered as a combination of steady (or mean or average) stress and a completely reversed stress component σ*v*. The following relations are derived from Fig. 2 (*e*):

5. Mean or average stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

6. Reversed stress component or alternating or variable stress,

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2}$$

For repeated loading, the stress varies from maximum to zero (*i.e.* $\sigma min = 0$) in each cycle as shown in Fig.2 (*d*).

$$\sigma_m = \sigma_v = \frac{\sigma_{max}}{2}$$

9. Stress ratio, $R = \sigma_{max}/\sigma_{min}$. For completely reversed stresses, $R = -1$ and for repeated stresses, $R = 0$. It may be noted that R cannot be greater than unity.

10.The following relation between endurance limit and stress ratio may be used

$$\sigma'_e = \frac{3\sigma_e}{2 - R}$$

<u>Effect of Loading on Endurance Limit—Load Factor</u>

The endurance limit (σ*e*) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let K_b = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.

K_a = Load correction factor for the reversed axial load. Its value may be taken as 0.8.

K_S = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

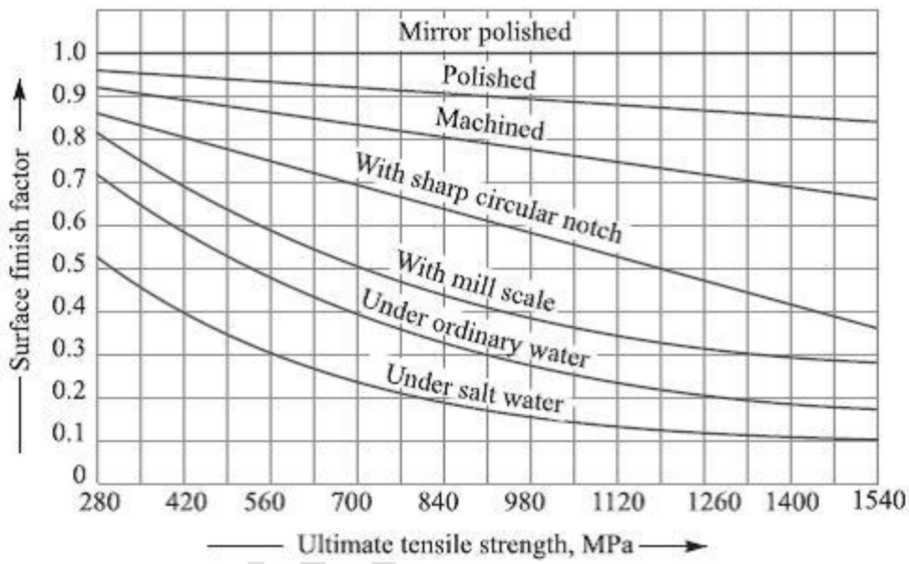
∴ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e K_b = \sigma_e$

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e K_a$

and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e K_s$

Effect of Surface Finish on En durance Limit—Surface Finish Factor

When a machine memb er is subjected to variable loads, the endurance limit of the material for that member depe nds upon the surface conditions. Fig. shows the values of surface finish factor for the vario us surface conditions and ultimate tensile stren gth.



When the surface finish factor is known, then the endurance limit for t he material of the machine member may be obtained by multiplying the endurance limit and the surface finish factor. We see that for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum a nd it goes on reducing due to surface conditio n.

Let K_{sur} = Surface finish fact or.

Then, Endurance limit,

$$\sigma_{e1} = \sigma_{eb}.K_{sur} = \sigma_e K_b.K_{sur} = \sigma_e.K_{sur} \quad ...(\because K_b = 1)$$

...(For reversed bending load)

$$= \sigma_{ea}.K_{sur} = \sigma_e.K_a.K_{sur} \quad ...(\text{For reversed axial load})$$

$$= \tau_e.K_{sur} = \sigma_e.K_s.K_{sur} \quad ...(\text{For reversed torsional or shear load})$$

Effect of Size on Endurance Limit—Size Factor

A little consideration will show that if the size of the standard specimen as shown in Fig.2 (*a*) is increased, then the endurance limit of the material will decrease. This is due to the fact that a longer specimen will have more defects than a smaller one. Let K_{sz} = Size factor.

Then, Endurance limit,

$$\sigma_{e2} = \sigma_{e1} \times K_{sz} \quad \text{...(Considering surface finish factor also)}$$
$$= \sigma_{eb}.K_{sur}.K_{sz} = \sigma_e.K_b.K_{sur}.K_{sz} = \sigma_e.K_{sur}.K_{sz} \quad (\because K_b = 1)$$
$$= \sigma_{ea}.K_{sur}.K_{sz} = \sigma_e.K_a.K_{sur}.K_{sz} \quad \text{...(For reversed axial load)}$$
$$= \tau_e.K_{sur}.K_{sz} = \sigma_e.K_s.K_{sur}.K_{sz} \quad \text{... (For reversed torsional or shear load)}$$

The value of size factor is taken as unity for the standard specimen having nominal diameter of 7.657 mm. When the nominal diameter of the specimen is more than 7.657 mm but less than 50 mm, the value of size factor may be taken as 0.85. When the nominal diameter of the specimen is more than 50 mm, then the value of size factor may be taken as 0.75.

Effect of Miscellaneous Factors on Endurance Limit

In addition to the surface finish factor (K_{sur}), size factor (K_{sz}) and load factors K_b, K_a and K_s, there are many other factors such as reliability factor (K_r), temperature factor (K_t), impact factor (K_i) etc. which has effect on the endurance limit of a material. Considering all these factors, the endurance limit may be determined by using the following expressions:

13. For the reversed bending load, endurance limit,

$$\sigma'_e = \sigma_{eb}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

14. For the reversed axial load, endurance limit,

$$\sigma'_e = \sigma_{ea}.K_{sur}.K_{sz}.K_r.K_t.K_i$$

15. For the reversed torsional or shear load, endurance limit,

$$\sigma'_e = \tau_e.K_{sur}.K_{sz}.K_r.K_t.K_i$$

In solving problems, if the value of any of the above factors is not known, it may be taken as unity.

Relation between Endurance Limit and Ultimate Tensile Strength

It has been found experimentally that endurance limit (σ_e) of a material subjected to fatigue loading is a function of ultimate tensile strength (σ_u).

Factor of Safety for Fatigue Loading

When a component is subjected to fatigue loading, the endurance limit is the criterion for failure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

For steel, $\sigma_e = 0.8 \text{ to } 0.9\ \sigma_y$

σ_e = Endurance limit stress for completely reversed stress cycle, and

σ_y = Yield point stress.

Fatigue Stress Concentration Factor

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as

Fatigue stress concentration factor,

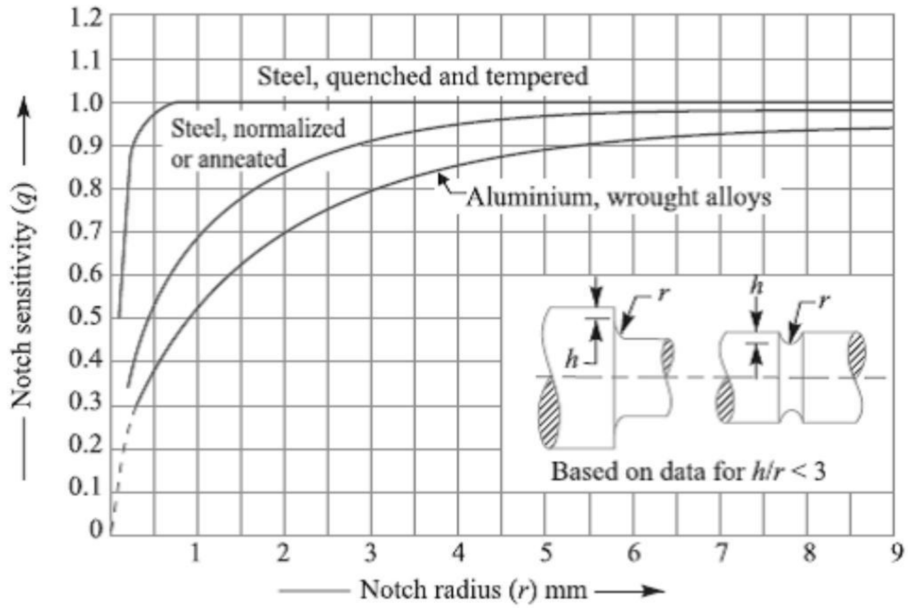
$$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

Notch Sensitivity

In cyclic loading, the effect of the notch or the fillet is usually less than predicted by the use of the theoretical factors as discussed before. The difference depends upon the stress gradient in the region of the stress concentration and on the hardness of the material. The term ***notch sensitivity*** is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material. Since the extensive data for estimating the notch sensitivity factor (*q*) is not available, therefore the curves, as shown in Fig., may be used for determining the values of *q* for two steals. When the notch sensitivity factor *q* is used in cyclic loading, then fatigue stress concentration factor may be obtained from the following relations:

$$q = \frac{K_f - 1}{K_t - 1}$$

Or



$$K_f = 1 + q\,(K_t - 1) \quad \text{...[For tensile or bending stress]}$$

And

$$K_{fs} = 1 + q\,(K_{ts} - 1) \quad \text{...[For shear stress]}$$

Where K_t = Theoretical stress concentration factor for axial or bending loading, and K_{ts} = Theoretical stress concentr ation factor for torsional or shear loading.

References:

6. Machine Design - V.Bandari .

7. Machine Design – R.S. Khurm i

8. Design Data hand Book - S MD Jalaludin.

Problem: Determine the thickness of a 120 mm wide uniform plate for sa fe continuous operation if the plate is to be sub jected to a tensile load that has a maximum vallue of 250 kN and a minimum value of 100 kN. The properties of the plate material are as follows: Endurance limit stress = 225 MPa, and Yield point stress = 300 MPa. The factor of safety based on yield point may be taken as 1.5.

Let t = Thickness of the plate in mm.

∴ Area, $A = b \times t = 120\,t$ mm^2

We know that mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{250 + 100}{2} = 175 \text{ kN} = 175 \times 10^3 \text{ N}$$

∴ Mean stress, $\sigma_m = \frac{W_m}{A} = \frac{175 \times 10^3}{120t}$ N/mm^2

Variable load, $W_v = \frac{W_{max} - W_{min}}{2} = \frac{250 - 100}{2} = 75 \text{ kN} = 75 \times 10^3$ N

∴ Variable stress, $\sigma_v = \frac{W_v}{A} = \frac{75 \times 10^3}{120t}$ N/mm^2

According to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{1.5} = \frac{175 \times 10^3}{120t \times 300} + \frac{75 \times 10^3}{120t \times 225} = \frac{4.86}{t} + \frac{2.78}{t} = \frac{7.64}{t}$$

∴ $t = 7.64 \times 1.5 = 11.46$ say 11.5 mm **Ans.**

Problem:

Determine the diameter of a circular rod made of ductile material with a fattigue strength (complete stress reversal), σ_e = 265 MPa and a tensile yield strength of 3 50 MPa. The member is subjected to a varyin g axial load from $W_{min} = -300 \times 10^3$ N to $W_{max} = 700 \times 10^3$ N and has a stress concentration factor = 1.8. Use factor of safety as 2.0.

Let d = Diameter of the circular rod in mm.

∴ Area, $A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2$ mm^2

We know that the mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{700 \times 10^3 + (-300 \times 10^3)}{2} = 200 \times 10^3 \text{ N}$$

∴ Mean stress, $\sigma_m = \frac{W_m}{A} = \frac{200 \times 10^3}{0.7854\, d^2} = \frac{254.6 \times 10^3}{d^2}$ N/mm^2

Variable load, $W_v = \frac{W_{max} - W_{min}}{2} = \frac{700 \times 10^3 - (-300 \times 10^3)}{2} = 500 \times 10^3$ N

$\therefore$ Variable stress, $\sigma_v = \frac{W_v}{A} = \frac{500 \times 10^3}{0.7854\, d^2} = \frac{636.5 \times 10^3}{d^2}$ N/mm²

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}$$

$$\frac{1}{2} = \frac{254.6 \times 10^3}{d^2 \times 350} + \frac{636.5 \times 10^3 \times 1.8}{d^2 \times 265} = \frac{727}{d^2} + \frac{4323}{d^2} = \frac{5050}{d^2}$$

$\therefore$ $d^2 = 5050 \times 2 = 10\,100$ or $d = 100.5$ mm **Ans.**

Problem:

A circular bar of 500 mm leng th is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maxi mum value of 50 kN. Determine the diameter of bar by taking a factor of safety of 1.5, size e ffect of 0.85, surface finish factor of 0.9. The material properties of bar are given by: ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa.

Solution. Given : $l = 500$ mm ; $W_{min} = 20$ kN $= 20 \times 10^3$ N ; $W_{max} = 50$ kN $= 50 \times 10^3$ N ; $F.S. = 1.5$; $K_{sz} = 0.85$; $K_{sur} = 0.9$; $\sigma_u = 650$ MPa $= 650$ N/mm² ; $\sigma_y = 500$ MPa $= 500$ N/mm² ; $\sigma_e = 350$ MPa $= 350$ N/mm²

Let d = Diameter of the bar in mm.

We know that the maximum bending moment,

$$M_{max} = \frac{W_{max} \times l}{4} = \frac{50 \times 10^3 \times 500}{4} = 6250 \times 10^3 \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times l}{4} = \frac{20 \times 10^3 \times 500}{4} = 2550 \times 10^3 \text{ N-mm}$$

$\therefore$ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{6250 \times 10^3 + 2500 \times 10^3}{2} = 4375 \times 10^3 \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{6250 \times 10^3 - 2500 \times 10^3}{2} = 1875 \times 10^3 \text{ N-mm}$$

Section modulus of the bar,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

$\therefore$ Mean or average bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{4375 \times 10^3}{0.0982\, d^3} = \frac{44.5 \times 10^6}{d^3} \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{1875 \times 10^3}{0.0982\, d^3} = \frac{19.1 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 650} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{68 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{139 \times 10^3}{d^3}$$

$\therefore \quad d^3 = 139 \times 10^3 \times 1.5 = 209 \times 10^3$ or $d = 59.3$ mm

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 500} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \quad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{89 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{160 \times 10^3}{d^3}$$

$\therefore \quad d^3 = 160 \times 10^3 \times 1.5 = 240 \times 10^3$ or $d = 62.1$ mm

Taking larger of the two values, we have $d = 62.1$ mm **Ans.**

References:

9. Machine Design - V.Bandari .

10.Machine Design – R.S. Khurm i

11.Design Data hand Book - S MD Jalaludin.

Problem:

A 50 mm diameter shaft is mad e from carbon steel having ultimate tensile st rength of 630 MPa. It is subjected to a torque which fluctuates between 2000 N-m to – 80 0 N-m. Using Soderberg method, calculate the factor of safety. Assume suitable values for any other data needed.

Solution. Given : $d = 50$ mm ; $\sigma_u = 630$ MPa $= 630$ N/mm^2 ; $T_{max} = 2000$ N-m ; $T_{min} = -800$ N-m

We know that the mean or average torque,

$$T_m = \frac{T_{max} + T_{min}}{2} = \frac{2000 + (-800)}{2} = 600 \text{ N-m} = 600 \times 10^3 \text{ N-mm}$$

∴ Mean or average shear stress,

$$\tau_m = \frac{16\,T_m}{\pi d^3} = \frac{16 \times 600 \times 10^3}{\pi (50)^3} = 24.4 \text{ N/mm}^2 \qquad \ldots \left(\because\ T = \frac{\pi}{16} \times \tau \times d^3\right)$$

Variable torque,

$$T_v = \frac{T_{max} - T_{min}}{2} = \frac{2000 - (-800)}{2} = 1400 \text{ N-m} = 1400 \times 10^3 \text{ N-mm}$$

$$\therefore \text{ Variable shear stress, } \tau_v = \frac{16\,T_v}{\pi d^3} = \frac{16 \times 1400 \times 10^3}{\pi (50)^3} = 57 \text{ N/mm}^2$$

Since the endurance limit in reversed bending (σ_e) is taken as one-half the ultimate tensile strength (*i.e.* $\sigma_e = 0.5\,\sigma_u$) and the endurance limit in shear (τ_e) is taken as 0.55 σ_e, therefore

$$\tau_e = 0.55\,\sigma_e = 0.55 \times 0.5\,\sigma_u = 0.275\,\sigma_u$$
$$= 0.275 \times 630 = 173.25 \text{ N/mm}^2$$

Assume the yield stress (σ_y) for carbon steel in reversed bending as 510 N/mm^2, surface finish factor (K_{sur}) as 0.87, size factor (K_{sz}) as 0.85 and fatigue stress concentration factor (K_{fs}) as 1.

Since the yield stress in shear (τ_y) for shear loading is taken as one-half the yield stress in reversed bending (σ_y), therefore

$$\tau_y = 0.5\,\sigma_y = 0.5 \times 510 = 255 \text{ N/mm}^2$$

Let $F.S.$ = Factor of safety.

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\tau_m}{\tau_y} + \frac{\tau_v \times K_{fs}}{\tau_e \times K_{sur} \times K_{sz}} = \frac{24.4}{255} + \frac{57 \times 1}{173.25 \times 0.87 \times 0.85}$$
$$= 0.096 + 0.445 = 0.541$$

$$\therefore \quad F.S. = 1 / 0.541 = 1.85 \text{ Ans.}$$

A simply supported beam has a concentrated load at the centre which fluctuates from a value of P to 4 P. The span of the bea m is 500 mm and its cross-section is circular with a diameter of 60 mm. Taking for the beam material an ultimate stress of 700 MPa, a yield stress of 500 MPa, endurance limit of 330 MPa for reversed bending, and a factor of s afety of 1.3, calculate the maximum value of P. Take a size factor of 0.85 and a surface finish factor of 0.9.

Solution. Given : $W_{min} = P$; $W_{max} = 4P$; $L = 500$ mm; $d = 60$ mm ; $\sigma_u = 700$ MPa $= 700$ N/mm^2 ; $\sigma_y = 500$ MPa $= 500$ N/mm^2 ; $\sigma_e = 330$ MPa $= 330$ N/mm^2 ; $F.S. = 1.3$; $K_{sz} = 0.85$; $K_{sur} = 0.9$

We know that maximum bending moment,

$$M_{max} = \frac{W_{max} \times L}{4} = \frac{4P \times 500}{4} = 500\,P \text{ N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times L}{4} = \frac{P \times 500}{4} = 125\,P \text{ N mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{500\,P + 125\,P}{2} = 312.5\,P \text{ N-mm}$$

and variable bending moment,

$$M_v = \frac{M_{max} - M_{min}}{2} = \frac{500\,P - 125\,P}{2} = 187.5\,P \text{ N-mm}$$

Section modulus, $$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(60)^3 = 21.21 \times 10^3 \text{ mm}^3$$

∴ Mean bending stress,

$$\sigma_m = \frac{M_m}{Z} = \frac{312.5\,P}{21.21 \times 10^3} = 0.0147\,P \text{ N/mm}^2$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{187.5\,P}{21.21 \times 10^3} = 0.0088\,P \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} - \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\,P}{700} + \frac{0.0088\,P \times 1}{330 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{21P}{10^6} + \frac{34.8\,P}{10^6} = \frac{55.8\,P}{10^6}$$

∴ $$P = \frac{1}{1.3} \times \frac{10^6}{55.8} = 13\,785 \text{ N} = 13.785 \text{ kN}$$

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.3} = \frac{0.0147\,P}{500} + \frac{0.0088\,P \times 1}{330 \times 0.9 \times 0.85} = \frac{29.4\,P}{10^6} + \frac{34.8\,P}{10^6} = \frac{64.2P}{10^6}$$

$$\therefore \quad P = \frac{1}{1.3} \times \frac{10^6}{64.2} = 11\,982 \text{ N} = 11.982 \text{ kN}$$

From the above, we find that maximum value of $P = 13.785$ kN **Ans.**

UNIT 2

Shafts:

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending.

In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used. An ***axle*,** though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave. A ***spindle*** is a short shaft that imparts motion either to a cutting tool (*e.g*. drill press spindles) or to a work piece (*e.g*. lathe spindles).

Types of Shafts

The following two types of shafts are important from the subject point of view:

1. *Transmission shafts*. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. *Machine shafts*. These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Stresses in Shafts

The following stresses are induced in the shafts:

R Shear stresses due to the transmission of torque (*i.e*. due to torsional load).

S Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

T Stresses due to combined torsional and bending loads.

Design of Shafts

The shafts may be designed on the basis of

2. Strength, and **2.** Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered:

(*a*) Shafts subjected to twisting moment or torque only,

(*b*) Shafts subjected to bending moment only,

(*c*) Shafts subjected to combined twisting and bending moments, and

(*d*) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts Subjected to Twisting Moment Only

a) Solid shaft:

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

1.

$$\overline{}\; J \quad r \; \overline{}$$

Where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre = d / 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J \; \frac{}{32} d^4$$

Then we get, $$T \; \frac{d^3}{16}$$

From this equation, diameter of the solid shaft (d) may be obtained.

b) Hollow Shaft:

We also know that for hollow shaft, polar moment of inertia,

$$J \quad \frac{(d_0)^4 \quad (d_i)^4}{32}$$

Where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_0 / 2$.

Substituting these values in equation **(*i*)**, we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$

Let k = Ratio of inside diameter and outside diameter of the shaft = d_i / d_o Now the equation **(*iii*)** may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

From the equations, the outside and inside diameter of a hollow shaft may be determined. It may be noted that

α The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft. When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3\,(1 - k^4) = d^3$$

1. The twisting moment (T) may be obtained by using the following relation: We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

Where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

mm In case of belt drives, the twisting moment (T) is given by

1.= $(T_1 - T_2)\,R$

Where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and R = Radius of the pulley.

<u>Shafts Subjected to Bending Moment Only</u>

<u>a) Solid Shaft:</u>

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

Where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation, σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (*d*) may be obtained.

b) Hollow Shaft:

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \quad \text{...(where } k = d_i / d_o\text{)}$$

And $y = d_0 / 2$

Again substituting these values in equation, we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (*do*) may be obtained.

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view:

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

a) Solid Shaft:

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of σ_b and τ

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

or $$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces **the same shear stress (τ) as the actual twisting moment. By limiting**

the maximum shear stress (τ_{max}) equal to the allowable shear stress (**τ**) for the material, the equation ***(i)*** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3$$

From this expression, diameter of the shaft (d*)* may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}(M + \sqrt{M^2 + T^2})\right]$$

or $$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right]$$

The expression $\frac{1}{2}[M + \sqrt{M^2 + T^2}]$ is known as ***equivalent bending moment*** and is denoted

by M_e. The equivalent bending moment may be defined as that moment which when acting alone produces the same tensile or compressive stress (**σ_b**) as the actual bending moment. By limiting the m**aximum normal stress [σ_b(*max*)] equal to the allowable bending stress (σ***b***)**, then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this expression, diameter of the shaft (d*)* may be evaluated.

<u>b) Hollow shaft:</u>

In case of a hollow shaft, the equations ***(ii)*** and ***(v)*** may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$$

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) - \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

References:

1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurmi

3. Design Data hand Book - S MD Jalaludin.

Problem:

A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another p ulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and **μ** = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in ten sion and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

Solution. Given : $AB = 800$ mm ; $\alpha_C = 20°$; $D_C = 600$ mm or $R_C = 300$ mm ; $AC = 200$ mm ; $D_D = 700$ mm or $R_D = 350$ mm ; $DB = 250$ mm ; $\theta = 180° = \pi$ rad ; $W = 2000$ N ; $T_1 = 3000$ N ; $T_1/T_2 = 3$; $\tau = 40$ MPa $= 40$ N/mm^2

The space diagram of the shaft is shown in Fig (*a*).

We know that the torque acting on the shaft at *D*,

$$T = (T_1 - T_2)\,R_D = T_1\left(1 - \frac{T_2}{T_1}\right)R_D$$

$$= 3000\left(1 - \frac{1}{3}\right)350 = 700 \times 10^3 \text{ N-mm} \qquad \ldots(\because T_1/T_2 = 3)$$

The torque diagram is shown in Fig. (*b*).

Assuming that the torque at *D* is equal to the torque at *C*, therefore the tangential force acting on the gear *C*,

$$F_{tc} = \frac{T}{R_C} = \frac{700 \times 10^3}{300} = 2333 \text{ N}$$

and the normal load acting on the tooth of gear *C*,

$$W_C = \frac{F_{tc}}{\cos\alpha_C} = \frac{2333}{\cos 20°} = \frac{2333}{0.9397} = 2483 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. Resolving the normal load vertically and horizontally, we get

Vertical component of W_C *i.e.* the vertical load acting on the shaft at *C*,

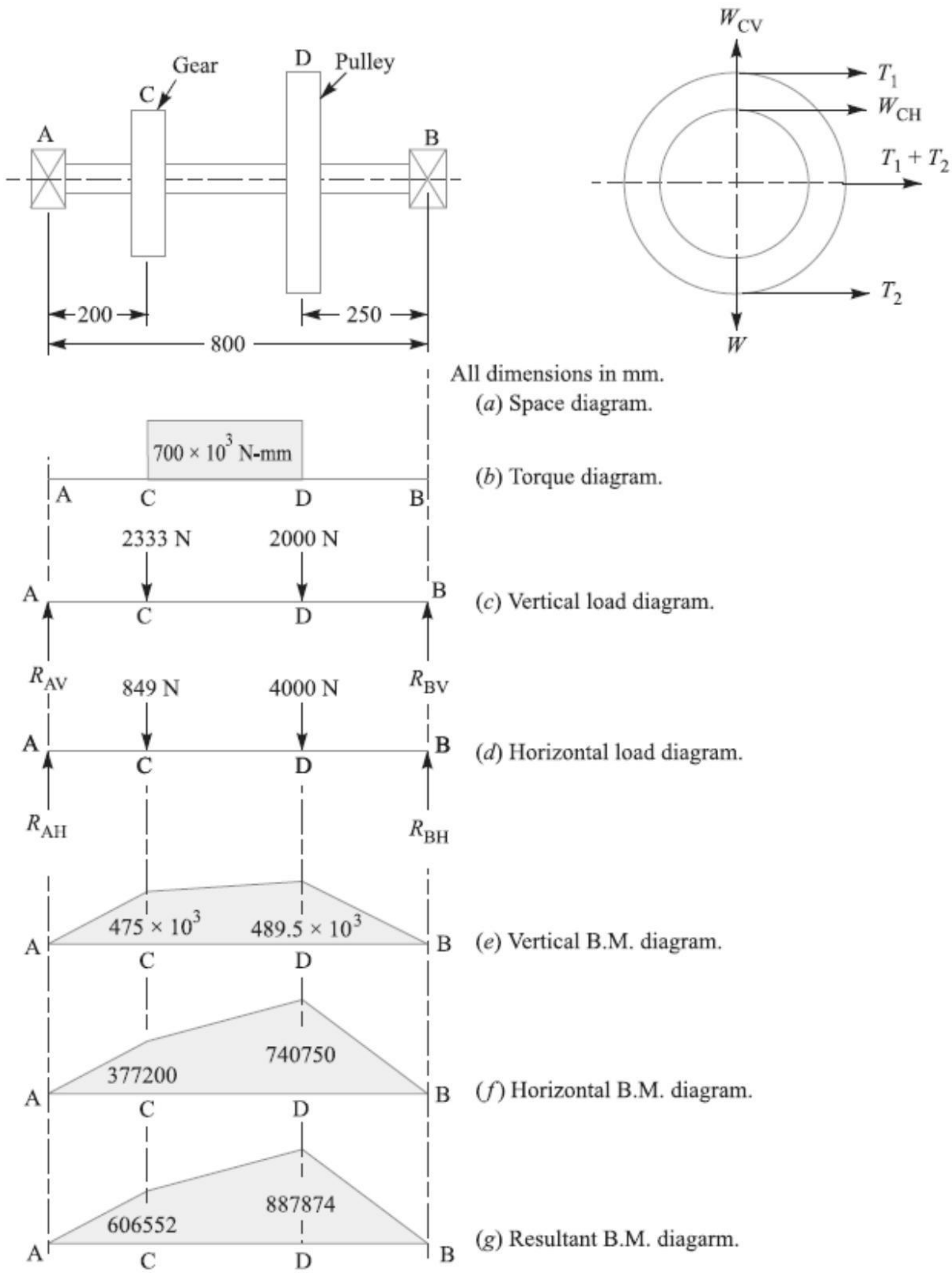
$$W_{CV} = W_C \cos 20°$$
$$= 2483 \times 0.9397 = 2333 \text{ N}$$

and horizontal component of W_C *i.e.* the horizontal load acting on the shaft at *C*,

$$W_{CH} = W_C \sin 20°$$
$$= 2483 \times 0.342 = 849 \text{ N}$$

Since $T_1 / T_2 = 3$ and $T_1 = 3000$ N, therefore

$$T_2 = T_1 / 3 = 3000 / 3 = 1000 \text{ N}$$



∴ Horizontal load acting on the shaft at D,

$$W_{DH} = T_1 + T_2 = 3000 + 1000 = 4000 \text{ N}$$

and vertical load acting on the shaft at D,

$$W_{DV} = W = 2000 \text{ N}$$

The vertical and horizontal load diagram
respectively.

Now let us find the maximum bending mome

First of all considering the vertical loading at bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 2333 + 2000 = 4$$

Taking moments about A, we get

$$R_{BV} \times 800 = 2000\ (800 - 250)$$
$$= 1\ 566\ 600$$

$\therefore$ $R_{BV} = 1\ 566\ 600 / 800 =$

and $R_{AV} = 4333 - 1958 = 2$

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $M_{CV} = R_{AV} \times 200 = 237$
$= 475 \times 10^3$ N-mm

B.M. at D, $M_{DV} = R_{BV} \times 250 = 195$

The bending moment diagram for vertical loa

Now consider the horizontal loading at C and D
A and B respectively. We know that

$$R_{AH} + R_{BH} = 849 + 4000 = 48$$

Taking moments about A, we get

$$R_{BH} \times 800 = 4000\ (800 - 250)$$

$\therefore$ $R_{BH} = 2\ 369\ 800 / 800 =$

and $R_{AH} = 4849 - 2963 = 1$

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $M_{CH} = R_{AH} \times 200 = 188$

B.M. at D, $M_{DH} = R_{BH} \times 250 = 296$

The bending moment diagram for horizontal l

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_C}$$
$$= 606\ 552 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_I}$$
$$= 887\ 874 \text{ N-mm}$$

Maximum bending moment

The resultant B.M. diagram is shown in Fig
maximum at D, therefore

Maximum B.M., $M = M_D = 887\ 874$ N-

The vertical and horizontal loa
respectively.
Now let us find the maximum ber
First of all considering the vertic
bearings A and B respectively. We kno
R_AV + R_BV = 2333
Taking moments about A, we get
R_BV × 800 = 2000
= 1 56
∴ R_BV = 1 56
and R_AV = 4333
We know that B.M. at A and B,
M_AV = M_BV
B.M. at C, M_CV = R_AV
= 475
B.M. at D, M_DV = R_BV
The bending moment diagram fo
Now consider the horizontal loadi
A and B respectively. We know that
R_AH + R_BH = 849
Taking moments about A, we get
R_BH × 800 = 4000
∴ R_BH = 2 36
and R_AH = 4849
We know that B.M. at A and B,
M_AH = M_BH
B.M. at C, M_CH = R_AH
B.M. at D, M_DH = R_BH
The bending moment diagram fo
We know that resultant B.M. at C
M_C = √(M
= 606
and resultant B.M. at D,
M_D = √(M
= 887
Maximum bending moment
The resultant B.M. diagram is s
maximum at D, therefore
Maximum B.M., M = M_D =

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Diameter of the shaft

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(887\,874)^2 + (700 \times 10^3)^2} = 1131 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1131 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 1131 \times 10^3 / 7.86 = 144 \times 10^3$ or $d = 52.4$ say 55 mm **Ans.**

Problem:

A steel solid shaft transmitting 15 kW at 200 r.p.m. is supported on two bearings 750 mm apart and has two gears keyed to it. The pinion having 30 teeth of 5 mm module is located 100 mm to the left of the right hand bearing and delivers power horizontally to the right. The gear having 100 teeth of 5 mm m odule is located 150 mm to the right of the left hand bearing and receives power in a vertical direction from below. Using an allowable stress of 54 MPa in shear, determine the diameter of the shaft.

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 200 r.p.m. ; AB = 750 mm ; T_D = 30 ; m_D = 5 mm ; BD = 100 mm ; T_C = 100 ; m_C = 5 mm ; AC = 150 mm ; τ = 54 MPa = 54 N/mm^2

The space diagram of the shaft is shown in Fig. 14.8 (*a*).

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 200} = 716 \text{ N-m} = 716 \times 10^3 \text{ N-mm}$$

The torque diagram is shown in Fig. 14.8 (*b*).

We know that diameter of gear

= No. of teeth on the gear × module

$\therefore$ Radius of gear C,

$$R_C = \frac{T_C \times m_C}{2} = \frac{100 \times 5}{2} = 250 \text{ mm}$$

and radius of pinion D,

$$R_D = \frac{T_D \times m_D}{2} = \frac{30 \times 5}{2} = 75 \text{ mm}$$

Assuming that the torque at C and D is same (*i.e.* 716 × 10^3 N-mm), therefore tangential force on the gear C, acting downward,

$$F_{tC} = \frac{T}{R_C} = \frac{716 \times 10^3}{250} = 2870 \text{ N}$$

and tangential force on the pinion D, acting horizontally,

$$F_{tD} = \frac{T}{R_D} = \frac{716 \times 10^3}{75} = 9550 \text{ N}$$

The vertical and horizontal load diagram is shown in Fig. 14.8 (*c*) and (*d*) respectively.

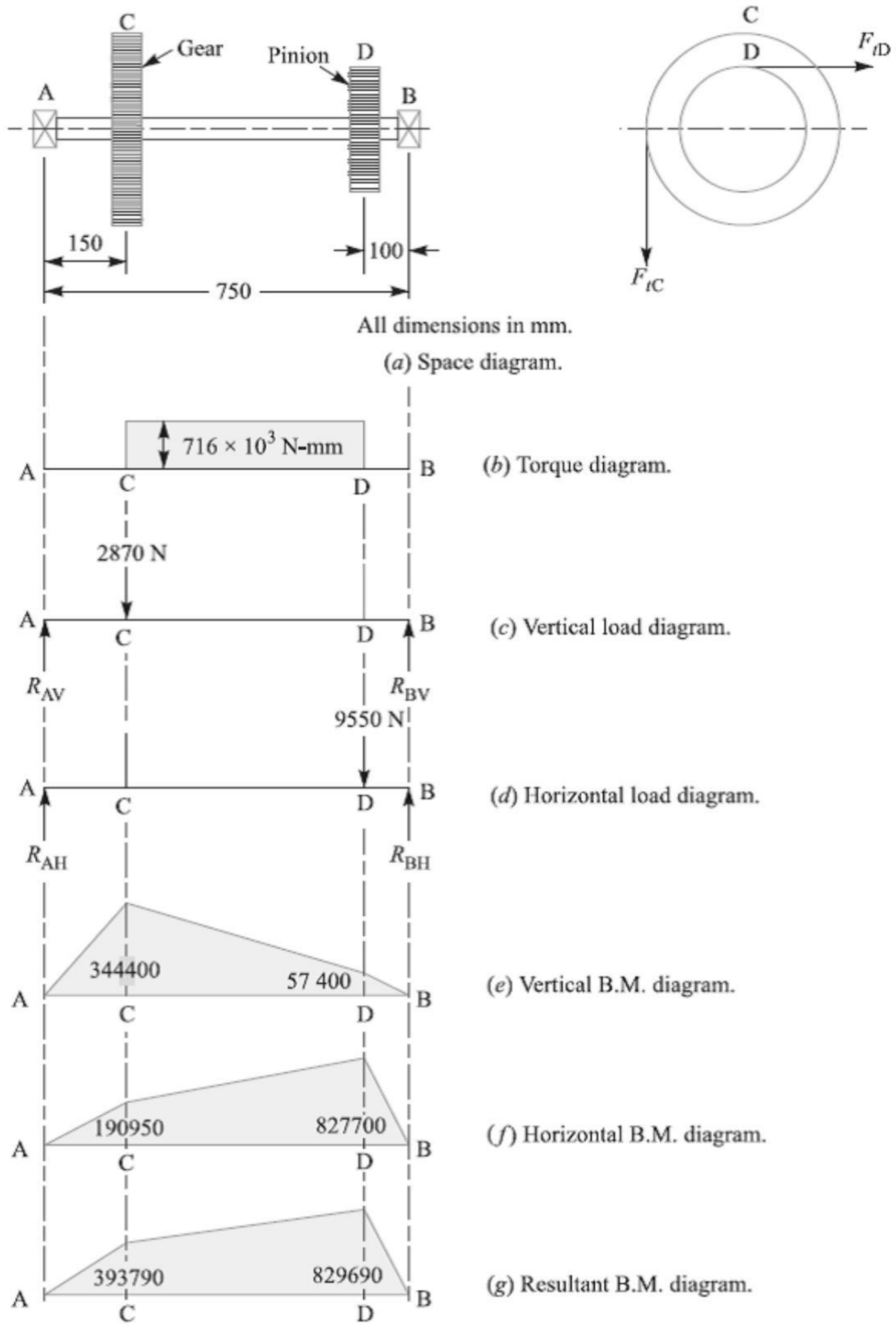
Now let us find the maximum bending moment for vertical and horizontal loading.

First of all, considering the vertical loading at *C*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AV} + R_{EV} = 2870 \text{ N}$$

Taking moments about *A*, we get

$$R_{BV} \times 750 - 2870 \times 150$$



$\therefore \quad R_{BV} = 2870 \times 150 / 750 = 574$ N

and $R_{AV} = 2870 - 574 = 2296$ N

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $\quad M_{CV} = R_{AV} \times 150 = 2296 \times 150 = 344\,400$ N-mm

B.M. at D, $\quad M_{DV} = R_{BV} \times 100 = 574 \times 100 = 57\,400$ N-mm

The B.M. diagram for vertical loading is shown in Fig. 14.8 (*e*).

Now considering horizontal loading at D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 9550 \text{ N}$$

Taking moments about A, we get

$$R_{BH} \times 750 = 9550\,(750 - 100) = 9550 \times 650$$

$\therefore \quad R_{BH} = 9550 \times 650 / 750 = 8277$ N

and $\quad R_{AH} = 9550 - 8277 = 1273$ N

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $\quad M_{CH} = R_{AH} \times 150 = 1273 \times 150 = 190\,950$ N-mm

B.M. at D, $\quad M_{DH} = R_{BH} \times 100 = 8277 \times 100 = 827\,700$ N-mm

The B.M. diagram for horizontal loading is shown in Fig. 14.8 (*f*).

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(344\,400)^2 + (190\,950)^2}$$
$$= 393\,790 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(57\,400)^2 + (827\,700)^2}$$
$$= 829\,690 \text{ N-mm}$$

The resultant B.M. diagram is shown in Fig. 14.8 (*g*). We see that the bending moment is maximum at D.

$\therefore$ Maximum bending moment,

$$M = M_D = 829\,690 \text{ N-mm}$$

Let $\quad d$ = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(829\,690)^2 + (716 \times 10^3)^2} = 1096 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1096 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 54 \times d^3 = 10.6\ d^3$$

$\therefore \quad d^3 = 1096 \times 10^3 / 10.6 = 103.4 \times 10^3$

or $\quad d = 47$ say 50 mm **Ans.**

REFERENCES:

1. Machine Design - V.Bandari .

4. Machine Design – R.S. Khurmi

5. Design Data hand Book - S MD Jalaludin.

Shafts Subjected to Axial Load in addition to Combined Torsion and Bennding Loads:

When the shaft is subjected to a n axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and sha fts for driving worm gears, then the stress due to axial load must be added to the bending stress (B). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M.y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32M}{\pi d^3}$$

And stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4F}{\pi d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \qquad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi (d_o)^2 (1 - k^2)} \qquad \text{... } (\because k = d_i/d_o)$$

Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32M}{\pi d^3} + \frac{4F}{\pi d^2} = \frac{32}{\pi d^3}\left(M + \frac{F \times d}{8}\right) \qquad \text{...}(i)$$

$$= \frac{32M_1}{\pi d^3} \qquad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32M}{\pi (d_o)^3 (1 - k^4)} + \frac{4F}{\pi (d_o)^2 (1 - k^2)}$$

$$= \frac{32}{\pi (d_o)^3 (1 - k^4)}\left[M + \frac{F d_o (1 + k^2)}{8}\right] = \frac{32M_1}{\pi (d_o)^3 (1 - k^4)}$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***COLUMN FACTOR* (α) must be introd uced to take the column effect into account.** Therefore, Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4F}{\pi d^2}$$

or

$$= \frac{\alpha \times 4F}{\pi (d_o)^2 \, (1 - k^2)}$$

The value of column factor (α) for compressive loads* may be obtained from the following relation :

Column factor,

$$\alpha - \frac{1}{1 - 0.0044 \, (L / K)}$$

This expression is used when the slenderness ratio (L / K) is less than 115. When the slenderness ratio (L / K) is more than 115, then the value of column factor may be obtained from the following relation:

Column factor, α

$$\alpha - \frac{\sigma_y \, (L / K)^2}{C \pi^2 \, E}$$

Where L = Length of shaft between the bearings,

K = Least radius of gyration,

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C =1, for hinged ends,

2.2.25, for fixed ends,

3.1.6, for ends that are partly restrained as in bearings.

In general, for a hollow shaft subjected to fluctuating torsional and bending load, along with an axial load, the equations for equivalent twisting moment (T_E) and equivalent bending moment (M_E) may be written as

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha \, F \, d_o \, (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4)$$

$$M_e = \frac{1}{2}\left[K_m \times M + \frac{\alpha F d_o \, (1 + k^2)}{8} + \sqrt{\left\{K_m \times M + \frac{\alpha F d_o \, (1 + k^2)}{8}\right\}^2 + (K_t \times T)^2}\right]$$

$$= \frac{\pi}{32} \times \sigma_b \, (d_o)^3 \, (1 - k^4)$$

It may be noted that for a solid shaft, K = 0 and $D_0 = D$. When the shaft carries no axial load, then F = 0 and **when the shaft carries axial tensile load, then α = 1.**

References:

3. Machine Design - V.Bandari .

4. Machine Design – R.S. Khurmi

5. Design Data hand Book - S MD Jalaludin.

Problem:

A hollow shaft is subjected to a maximum torque of 1.5 kN-m and a maxi mum bending moment of 3 kN-m. It is subjected, at the same time, to an axial load of 10 kN. Assume that the load is applied gradually and the ratio of the inner diameter to the outer diam eter is 0.5. If the outer diameter of the shaft is 80 mm, find the shear stress induced in the shaft.

SOLUTION. Given: $T = 1.5$ kN-m $= 1.5 \times 10^3$ N-m ; $M = 3$ kN-m $= 3 \times 10^3$ N-m ; $F = 10$ kN $= 10 \times 10^3$ N ; $k = d_i / d_o = 0.5$; $d_o = 80$ mm $= 0.08$ m

Let **τ = Shear stress induced in th e shaft.**

Since the load is applied gradually, therefore from DDB, we find that $K_m = 1.5$; and $K_t = 1.0$ We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1+k^2)^2}{8}\right] + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 3 \times 10^3 + \frac{1 \times 10 \times 10^3 \times 0.08 (1+0.5^2)^2}{8}\right] + (1 \times 1.5 \times 10^3)^2}$$

$$= \sqrt{(4500+125)^2 + (1500)^2} = 4862 \text{ N-m} = 4862 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$4862 \times 10^3 = \frac{\pi}{16} \times \tau (d_o)^3 (1-k^4) = \frac{\pi}{16} \times \tau (80)^3 (1-0.5^4) = 94\,260\,\tau$$

$$\therefore \quad \tau = 4862 \times 10^3 / 94\,260 = 51.6 \text{ N/mm}^2 = 51.6 \text{ MPa } \textbf{Ans.}$$

Problem:

A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used to drive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart an d it transmits 5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN.

Determine:

ω The maximum shear stress de veloped in the shaft, and

ω The angular twist between the bearings.

Solution. Given : $d_o = 0.5$ m ; $d_i = 0.3$ m ; $P = 5600$ kW $= 5600 \times 10^3$ W ; $L = 6$ m ; $N = 150$ r.p.m. ; $F = 500$ kN $= 500 \times 10^3$ N ; $W = 70$ kN $= 70 \times 10^3$ N

1. *Maximum shear stress developed in the shaft*

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{5600 \times 10^3 \times 60}{2\pi \times 150} = 356\ 460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52\ 500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2][(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

$\therefore$ Slenderness ratio,

$$L / K = 6 / 0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \quad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also $\quad k = d_i / d_o = 0.3 / 0.5 = 0.6$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o\ (1 + k^2)}{8}\right]^2 \ \ (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 52\ 500 + \frac{1.22 \times 500 \times 10^3 \times 0.5\ (1 + 0.6^2)}{8}\right]^2 + (1 \times 356\ 460)^2}$$

$$= \sqrt{(78\ 750 + 51\ 850)^2 + (356\ 460)^2} = 380 \times 10^3 \text{ N-m}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$380 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\ (1 - k^4) = \frac{\pi}{16} \times \tau\,(0.5)^3\ [1 - (0.6)^4] = 0.02\ \tau$$

$\therefore \quad \tau = 380 \times 10^3 / 0.02 = 19 \times 10^6$ N/m^2 = 19 MPa **Ans.**

2. ***Angular twist between the bearings***

Let θ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(0.5)^4 - (0.3)^4] = 0.005\,34 \text{ m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}, \text{ we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\,460 \times 6}{84 \times 10^9 \times 0.00\,534} = 0.0048 \text{ rad}$$

... (Taking $G = 84$ GPa $= 84 \times 10^9$ N/m^2)

$$= 0.0048 \times \frac{180}{\pi} = 0.275° \textbf{ Ans.}$$

REFERENCES:

1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurm i

3. Design Data hand Book - S MD Jalaludin.

Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

2. Torsional rigidity. The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be affected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft. The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G . \theta}{L} \quad \text{or} \quad \theta = \frac{T . L}{J . G}$$

where θ = Torsional deflection or angle of twist in radians, T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation, G = Modulus of rigidity for the shaft material, and L = Length of the shaft.

3. Lateral rigidity. It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *I.E.*

$$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

BIS codes of Shafts

The standard sizes of transmission shafts are:

25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps ; 110 mm to 140

1. with 15 mm steps ; and 140 mm to 500 mm with 20 mm steps. The standard length of the shafts are 5 m, 6 m and 7 m.

:

References:

2. Machine Design - V.Bandari .

3. Machine Design – R.S. Khurmi

4. Design Data hand Book - S MD Jalaludin.

Problem:

A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.

Solution. Given : $P = 4$ kW = 4000 W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m = 1000 mm ; $G = 84$ GPa $= 84 \times 10^9$ N/m² $= 84 \times 10^3$ N/mm²

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\,740 \text{ N-mm}$$

We also know that

$$\frac{T}{J} = \frac{G \times \theta}{L} \quad \text{or} \quad J = \frac{T \times l}{G \times \theta}$$

or

$$\frac{\pi}{32} \times d^4 = \frac{47\,740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\,167$$

$$\therefore \quad d^4 = 129\,167 \times 32 / \pi = 1.3 \times 10^6 \quad \text{or} \quad d = 33.87 \text{ say } 35 \text{ mm} \quad \textbf{Ans.}$$

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\,740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\,(35)^3 = 8420\,\tau$$

$$\therefore \quad \tau = 47\,740 / 8420 = 5.67 \text{ N/mm}^2 = 5.67 \text{ MPa} \quad \textbf{Ans.}$$

REFERENCES:

1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurm i

3. Design Data hand Book - S MD Jalaludin.

Introduction

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and ar e subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys

The following types of keys are important from the subject point of view :

4. Sunk keys, **2.** Saddle keys, **3.** Tangent keys, **4.** Round keys, and **5.** Splines.

Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyw ay of the hub or boss of the pulley. The sunk keys are of the following types :

7. Rectangular sunk key. A recctangular sunk key is shown in Fig. The usual proportions of this key are :

Width of key, $w = d / 4$; and thi ckness of key, $t = 2w / 3 = d / 6$
where d = Diameter of the shaft or diameter of the hole in the
hub. The key has taper 1 in 100 on th e top side only.

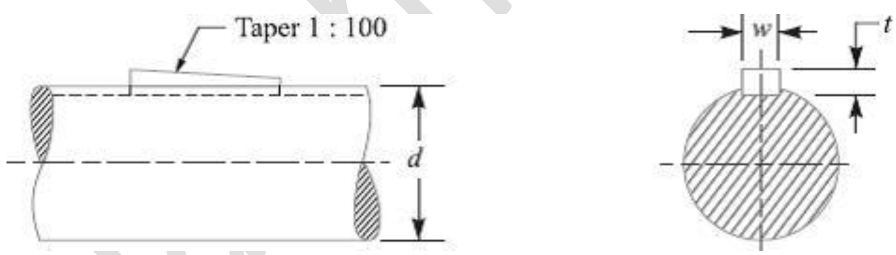


Fig. Sunk Key

11.Square sunk key. The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, i.e. $w = t = d / 4$

12.Parallel sunk key. The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taper less and is used where the pulley, gear or other mating piece is required to slide along the shaft.

13.Gib-head key. It is a rectangular sunk key with a head at one end known as **gib head.** It is usually provided to facilitate the removal of key. A gib head key is shown i n Fig.

16. and its use in shown in Fiig. (b).

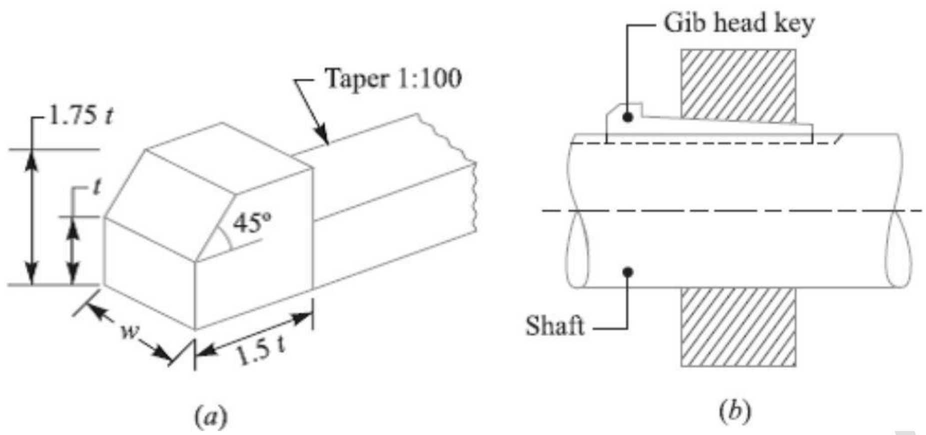


Fig. Gib head key and its use.

The usual proportions of the gib head key are:

Width, w = d / 4 ; and thickness at large end, t = 2w / 3 = d / 6

9. Feather key. A key attached to one member of a pair and which permits relative axial movement is known as **feather key.** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the sh aft or hub, the key being a sliding fit in the key way of the moving piece.

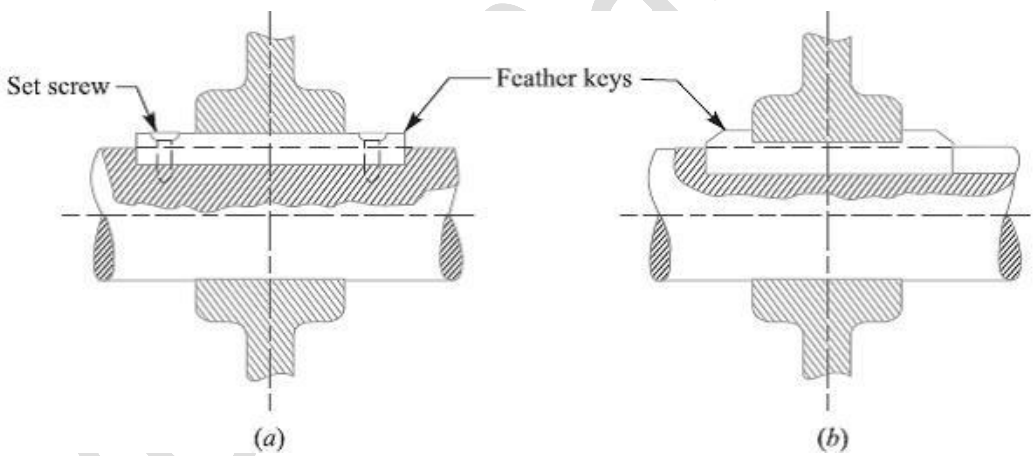


Fig. Feather Keys

12.Woodruff key. The woodr uff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

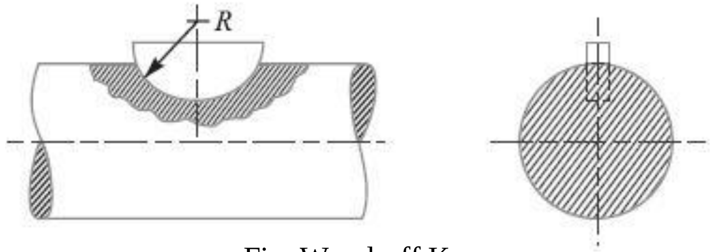


Fig. Woodruff Key

The main advantages of a woodruff key are as follows:

3. It accommodates itself to any taper in the hub or boss of the mating piece.

4. It is useful on tapering shaft ends. Its extra depth in the shaft prevents any ten dency to turn over in its keyway.

The disadvantages are:

(b) The depth of the keyway weak ens the shaft.

(c) It can not be used as a feather.

<u>Saddle keys</u>

The saddle keys are of the follow ing two types:

4. Flat saddle key, and **2.** Hollow saddle key.

A **flat saddle key** is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Fig. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

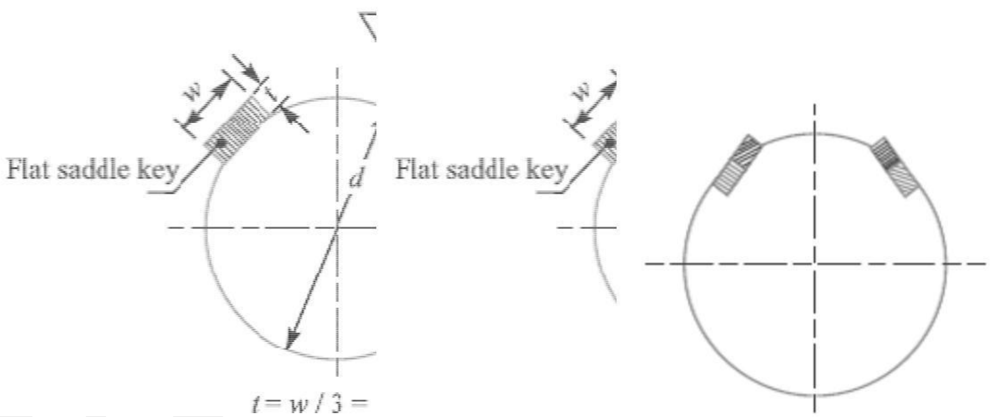


Fig. Flat saddle key and Tangent keys

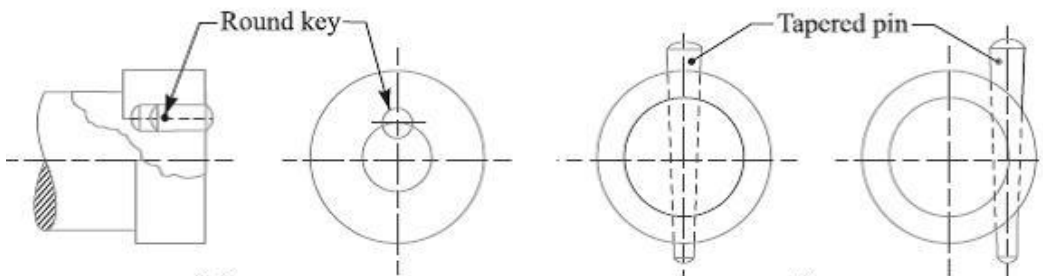
A **hollow saddle key** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

Tangent Keys

The tangent keys are fitted in p air at right angles as shown in Fig. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

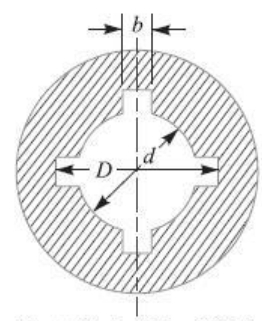
Round Keys

The round keys, as shown in Fig . (a) are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways m ay be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.



Splines

Sometimes, keys are made integgral with the shaft which fits in the keyways broached in the hub. Such shafts are known as **splined shafts** as shown in Fig . These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.



$D = 1.25\ d$ and $b = 0.25\ D$

Stresses in Keys:

Forces acting on a Sunk Key

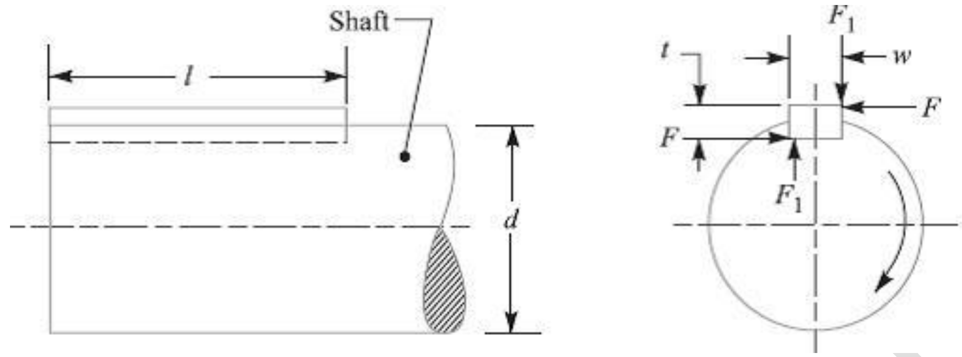
When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key:

3. Forces (F1) due to fit of the key in its keyway, as in a tight fitting straigh t key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

4. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig.

In designing a key, forces due to fit of the key are neglected and it is assu med that the distribution of forces along the leength of key is uniform.



<u>Strength of a Sunk Key</u>

A key connecting the shaft and hub is shown in Fig.

Let T = Torque transmitted by the shaft,

F = Tangential force acti ng at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key.

t = Thickness of key, and

10. and σc = Shear and cru shing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft,, the key may fail due to shearing or crushing. Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

F = Area resisting shearing × Shear stress = l × w ×

τ Therefore, Torque transmitted byy the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \quad ...(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

F = Area resisting crushing × Crushing stress

$$= l \times \frac{t}{2} \times \sigma_c$$

Therefore, Torque transmitted byy the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad ...(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

Or

$$\frac{w}{t} = \frac{\sigma_c}{2\tau}$$

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress. Therefore from the above equation, we have w = t. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft. We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2}$$

And torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3$$

From the above

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau}$$

When the key material is same as that of the shaft, then $\tau = \tau_1$. So, l = 1.571 d.

<u>Shaft Coupling</u>

Shafts are usually available up to 7 meters length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

Shaft couplings are used in machinery for several purposes, the most common of which are the following:

1. To provide for the connection of shafts of units those are manufactured separately such as a motor and generator and to provide for disconnection for repairs or alternations.

2. To provide for misalignment of the shafts or to introduce mechanical flexibility.

3. To reduce the transmission of shock loads from one shaft to another.

4. To introduce protection against overloads.

5. It should have no projecting parts.

Types of Shafts Couplings

Shaft couplings are divided into two main groups as follows:

3. Rigid coupling. It is used to connect two shafts which are perfectly aligned. Following types of rigid coupling are important from the subject point of view:

(*a*) Sleeve or muff coupling.

(*b*) Clamp or split-muff or compression coupling,

and **(*c*)** Flange coupling.

4. Flexible coupling. It is used to connect two shafts having both lateral and angular misalignment. Following types of flexible coupling are important from the subject point of view:

(*a*) Bushed pin type coupling,

(*b*) Universal coupling,

and **(*c*)** Oldham coupling.

Sleeve or Muff-coupling

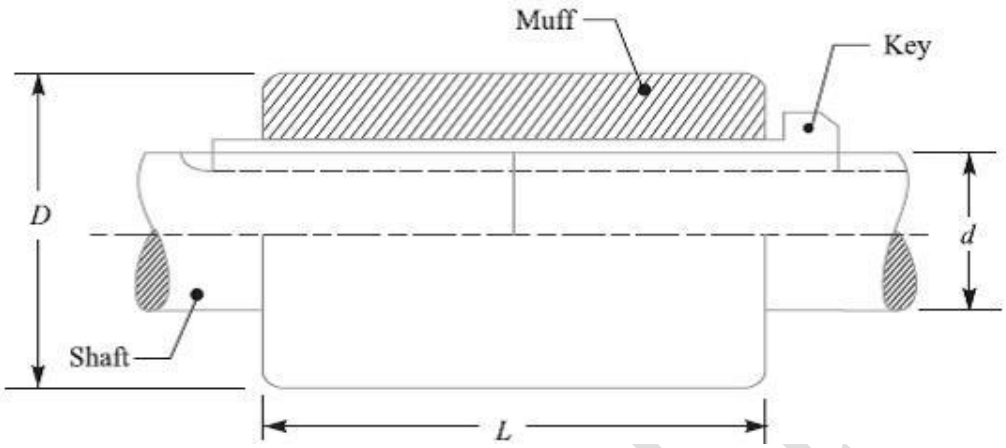
It is the simplest type of rigid coupling, made of cast iron. It consists of a hollow cylinder whose inner diameter is the same as that of the shaft. It is fitted over the ends of the two shafts by means of a gib head key, as shown in Fig. The power is transmitted from one shaft to the other shaft by means of a key and a sleeve. It is, therefore, necessary that all the elements must be strong enough to transmit the torque. The usual proportions of a cast iron sleeve coupling are as follows:

Outer diameter of the sleeve, $D = 2d + 13$ mm

And length of the slee ve, L = 3.5 d

Where d is the diameter of the shaft.

In designing a sleeve or muff-coupling, the following procedure may be adopted.



1. Design for sleeve

The sleeve is designed by considering it as a hollow shaft

Let T = Torque to be transmitted by the coupling, and

τ_c = Permissible shear stress for the material of the sleeve which is cast ir on.

The safe value of shear stress for cast iron may be taken as 14 MPa.

We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 \, (1 - k^4) \qquad \ldots (\because k = d/D)$$

From this expression, the induced shear stress in the sleeve may be checked.

2. Design for key

The key for the coupling may be designed in the similar way as discussed i n Unit-5. The width and thickness of the coupling key is obtained from the proportions. The length of the coupling key is at least equal to the length of the sleeve (i.e. 3.5 d). The coupling key is usually made into two parts so th at the length of the key in each shaft,

$$l = \frac{L}{2} = \frac{3.5\, d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that torqu e transmitted,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \ldots \text{(Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \ldots \text{(Considering crushing of the key)}$$

Note: The depth of the keyway in each of the shafts to be connected should be exactly the same and the diameters should also be same. If these conditions are not satisfied, then the key will be bedded on one shaft while in the other it will be loose. In order to prevent this, the key is made in two parts which may be driven from the same end for each shaft or they may be driven from opposite ends.

References:

1. Machine Design - V.Bandari .
2. Machine Design – R.S. Khurmi
3. Design Data hand Book - S MD Jalaludin.

-

Problem: Design and make a ne at dimensioned sketch of a muff coupling which is used to connect two steel shafts transmit ting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Solution.

Given: P = 40 kW = 40 × 10^3 W ; N = 350 r.p.m.; τ_s = 40 MPa = 40 N/mm2; $\sigma_{c\ s}$ = 80 MPa = 80 N/mm^2; σ_c = 15 MPa = 15 N/mm^2.

$$T - \frac{P \times 60}{2\,\pi N} - \frac{40 \times 10^3 \times 60}{2\,\pi \times 350} - 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 1100 \times 10^3/7.86 = 140 \times 10^3 \text{ or } d = 52 \text{ say } 55 \text{ mm Ans.}$$

2. Design for sleeve

We know that outer diameter of the muff,

D = 2d + 13 mm = 2 × 55 + 13 = 123 say 125 mm **Ans.**

and length of the muff,

L = 3.5 d = 3.5 × 55 = 192.5 say 195 mm **Ans.**

Let us now check the induced shear stress in the muff. Let τ_c be the induced shea r stress in the muff which is made of cast ir on. Since the muff is considered to be a hollow shaft, therefore the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$

$$= 370 \times 103\ \tau_c$$

$$\therefore \quad \tau_c = 1100 \times 10^3/370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm2, therefore the desi gn of muff is safe.

3. Design for key

From Design data Book, we find that for a shaft of 55 mm diameter,

Width of key, w = 18 mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

Then, Thickness of key, t = w = 18 mm **Ans.**

We know that length of key in each shaft,

l = L / 2 = 195 / 2 = 97.5 mm **Ans.**

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3\ \tau_s$$

$$\tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (*T*),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3\ \sigma_{cs}$$

$$\sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.

References:

1. Machine Design - V.Bandari .
2. Machine Design – R.S. Khurmi
3. Design Data hand Book - S MD Jalaludin.

Clamp or Compression Coupling or split muff coupling

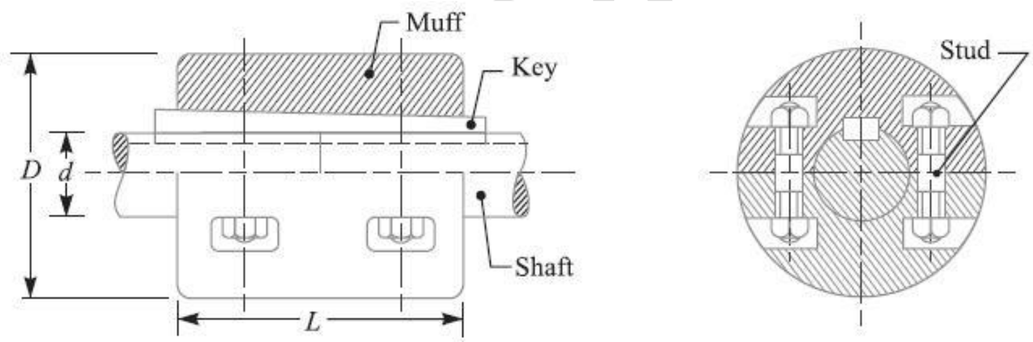
It is also known as **split muff coupling.** In this case, the muff or sleeve is m ade into two halves and are bolted together as shown in Fig. The halves of the muff are made of cast iron. The shaft ends are made to a butt each other and a single key is fitted directly in the keyways of both the shafts. One-half of th e muff is fixed from below and the other half is placed from above. Both the halves are held together by means of mild steel studs or bolts and nuts. The number of bolts may be two, four or six. The nuts are recessed into the bodies of the muff castings. This coupling may be used for heavy duty and moderate speeds. The advantage of this coupling is that the position of the shafts need not be changed for assembling or disassembling of the coupling. The usual proportions of the muff for the clamp or compression coupling are:

Diameter of the muff or s leeve, D = 2d + 13 mm

Length of the muff or sleeve, L = 3.5 d

Where d = Diameter of the shaft.

In the clamp or compres sion coupling, the power is transmitted from on e shaft to the other by means of key and the friction between the muff and shaft. In designing this type of coupling, the following procedure may be adopted.



1. Design of muff and key

The muff and key are designed i n the similar way as discussed in muff coupling.

2. Design of clamping bolts

Let T = Torque transmitted by the shaft,

d = Diameter of shaft,

d_b = Root or effective diameter of bolt,

n = Number of bolts,

σ_t = Permissible tensile stress for bolt material,

μ = Coefficient of frictio n between the muff and shaft, and

L = Length of muff.

We know that the force exerted by each bolt

$$= \frac{\pi}{4} (d_b)^2 \sigma_t$$

Then, Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d}$$

Then, Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

$$= \mu \times \frac{\frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d} \times \frac{1}{2} \pi d \times L$$

$$= \mu \times \frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2} \times \pi = \mu \times \frac{\pi^2}{8} (d_b)^2 \sigma_t \times n$$

And the torque that can be transmitted by the coupling,

$$T = F \times \frac{d}{2} = \mu \times \frac{\pi^2}{8} (d_b)^2 \sigma_t \times n \times \frac{d}{2} = \frac{\pi^2}{16} \times \mu \, (d_b)^2 \sigma_t \times n \times d$$

From this relation, the root diameter of the bolt (d_b) may be evaluated.

Flange Coupling

A flange coupling usually applies to a coupling having two separate cast iron flanges. Each flange is mounted on the shaft end and keyed to it. The faces are turned up at right angle to the axis of the shaft. One of the flanges has a projected portion and the other flange has a corresponding recess. This helps to bring the shafts into line and to maintain alignment. The two flanges are coupled together by means of bolts and nuts. The flange coupling is adapted to heavy loads and hence it is used on large shafting. The flange couplings are of the following three types:

σ **Unprotected type flange co upling**. In an unprotected type flange coupling, as shown in Fig.1, each shaft is keyed to the boss of a flange with a counter sunk key and t he flanges are coupled together by means of bo lts. Generally, three, four or six bolts are used. The keys are staggered at right angle along the circumference of the shafts in order t o divide the weakening effect caused by key ways.

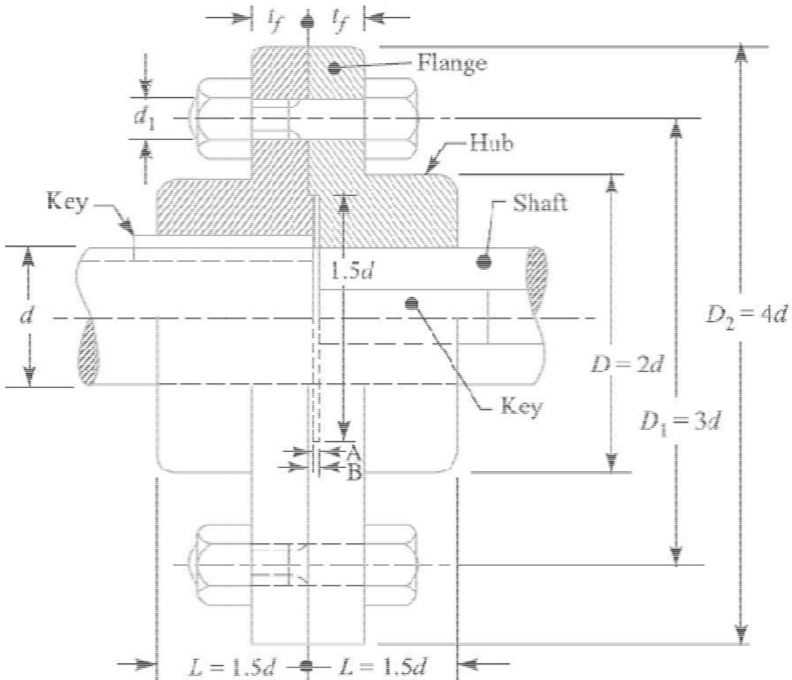


Fig.1 Unprotected Type Flange Coupling.

The usual proportions for an unprotected type cast iron flange couplings, as sho wn in Fig.1, are as follows:

If d is the diameter of the shaft or inner diameter of the hub, then Outside diameter of hub,

$D = 2\ d$

Length of hub, $L = 1.5\ d$

Pitch circle diameter of bolts, $D_1 = 3d$

Outside diameter of flange,

$D_2 = D_1 + (D_1 - D) = 2D_1 - D = 4\ d$

Thickness of flange, $t_f = 0.5\ d$

Number of bolts = 3, for d upto 40 mm

= 4, for d upto 100 mm

= 6, for d upto 180 mm

1. Protected type flange coupling. In a protected type flange coupling, as shown in Fig.2, the protruding bolts and nuts are protected by flanges on the two halves of th e coupling, in order to avoid danger to the workman. The thickness of the protective circumfe rential flange (t_p) is taken as 0.25 d. The other proportions of the coupling are same as for un protected type flange coupling.

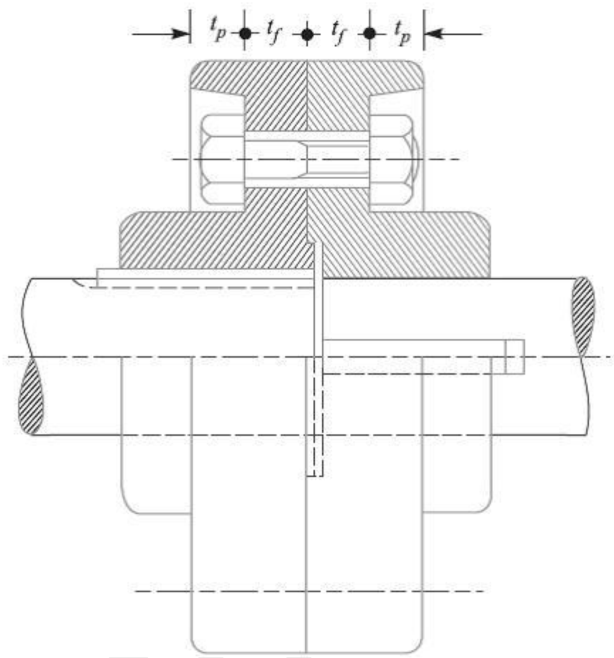


Fig.2 . Protected Type Flange Coupling.

= **Marine type flange coupling.** In a marine type flange coupling, the flanges are forged integral with the shafts as shown in Fig.3.

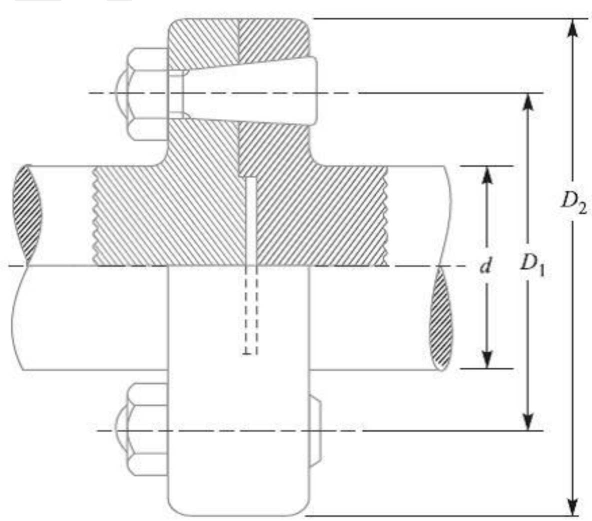


Fig.3. Solid Flange Coupling or Marine Type flange coupling.

The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft. The other proportions for the marine type flange coupling are taken as follows:

Thickness of flange = d / 3

Taper of bolt = 1 in 20 to 1 in 40

Pitch circle diameter of bolts, D_1 = 1.6 d

Outside diameter of flange, D_2 = 2.2 d

Design of Flange Coupling

Consider a flange coupling as shown in Fig.1 and Fig.2.

Let d = Diameter of shaft or inner diameter of hub,

D = Outer diameter of hub,

D_1 = Nominal or outside diameter of bolt,

D_1 = Diameter of bolt circle,

n = Number of bolts,

t_f = Thickness of flange,

τ_s, τ_b and τ_k = Allowable shear stress for shaft, bolt and key material

respectively τ_c = Allowable shear stress for the flange material i.e. cast iron,

σ_{cb}, and σ_{ck} = Allowable crushing stress for bolt and key material respectively.

The flange coupling is designed as discussed below:

1. Design for hub

The hub is designed by considering it as a hollow shaft, transmitting the same torque (T) as that of a solid shaft.

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

The outer diameter of hub is usually taken as twice the diameter of shaft. Therefore from the above relation, the induced shearing stress in the hub may be checked.

The length of hub (L) is taken as 1.5 d.

2. Design for key

The key is designed with usual proportions and then checked for shearing and crushing stresses. The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub.

3. Design for flange

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the torque transmitted,

T = Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

$$= \pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickness of flange is usually taken as half the diameter of shaft. Therefore from the above relation, the induced shearing stress in the flange may be checked.

4. Design for bolts

The bolts are subjected to shear stress due to the torque transmitted. The number of bolts (n) depends upon the diameter of shaft and the pitch circle diameter of bolts (D_1) is taken as 3 d. We know that

Load on each bolt

$$= \frac{\pi}{4} (d_1)^2 \ \tau_b$$

Then, Total load on all the bolts

$$= \frac{\pi}{4} (d_1)^2 \ \tau_b \times n$$

And torque transmitted,

$$T = \frac{\pi}{4} (d_1)^2 \ \tau_b \times n \times \frac{D_1}{2}$$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts = $n \times d_1 \times$

t_f And crushing strength of all the bolts = $(n \times d_1 \times t_f)\ \sigma_{cb}$

Torque,

$$T = (n \times d_1 \times t_f \times \sigma_{cb}) \ \frac{D_1}{2}$$

From this equation, the induced crushing stress in the bolts may be checked.

References:

1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurmi

3. Design Data hand Book - S MD Jalaludin.

Problem: Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a co mpressor. The service factor may be assumed as 1.35. The following permissible stresses may be used :

Shear stress for shaft, bolt and k ey material = 40

MPa Crushing stress for bolt and key = 80 MPa

Shear stress for cast iron = 8 MPa

Draw a neat sketch of the coupli ng.

Solution. Given: P = 15 kW = 15 × 103 W; N = 900 r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k$ 1.40 MPa = 40 N/mm^2 ; $\sigma_{cb} = \sigma_{ck}$ = 80 MPa = 80 N/mm^2 ; τ_c = 8 MPa = 8 N/mm^2. The protective type flange coupling is designed as discussed below:

1. Design for hub

First of all, let us find the diam eter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

T_{max} = 1.35 × 159.13 = 215 N-m = 215 × 103 N-mm

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3 \quad \text{or} \quad d = 30.1 \text{ say } 35 \text{ mm } \textbf{Ans.}$$

We know that outer diameter of the hub,

D = 2d = 2 × 35 = 70 mm Ans.

And length of hub, L = 1.5 d = 1.5 × 35 = 52.5 mm Ans.

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\,147\, \tau_c$$

Then, τ_c = 215 × 103/63 147 = 3.4 N/mm2 = 3.4 MPa

Since the induced shear stress fo r the hub material (i.e. cast iron) is less than th e permissible value of 8 MPa, therefore the de sign of hub is safe.

2. Design for key

Since the crushing stress for the key material is twice its shear stress (i.e. $\sigma_{ck} = 2\tau_k$),

therefore a square key may be used. From DDB, we find that for a shaft of 35 mm diameter,

Width of key, w = 12 mm Ans.

And thickness of key, t = w = 12 mm Ans.

The length of key (l) is taken equal to the length of hub.

Then, l = L = 52.5 mm Ans.

Let us now check the induced stresses in the key by considering it in shearing and crushing.
Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 - l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} - 11\,025\ \tau_k$$

Then, τ_k = 215 × 103/11 025 = 19.5 N/mm2 = 19.5 MPa

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

Σ_{ck} = 215 × 103/ 5512.5 = 39 N/mm^2 = 39 MPa.

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. Design for flange

The thickness of flange (t_f) is taken as 0.5 d.

Then, t_f = 0.5 d = 0.5 × 35 = 17.5 mm Ans.

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\,713\ \tau_c$$

τ_c = 215 × 103/134 713 = 1.6 N/mm2 = 1.6 MPa

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. Design for bolts

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

n = 3 and pitch circle diameter of bolts,

D_1 = 3d = 3 × 35 = 105 mm

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4}(d_1)^2 \ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2 \ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$(d_1)^2$ = 215 × 103/4950 = 43.43 or d_1 = 6.6

mm Assuming coarse threads, the nearest standard size of bolt is M 8. Ans. Other proportions of the flange are taken as follows:

Outer diameter of the flange,

D_2 = 4 d = 4 × 35 = 140 mm Ans.

Thickness of the protective circumferential flange,

t_p = 0.25 d = 0.25 × 35 = 8.75 say 10 mm Ans.

References:

= Machine Design - V.Bandari .

= Machine Design – R.S. Khurmi

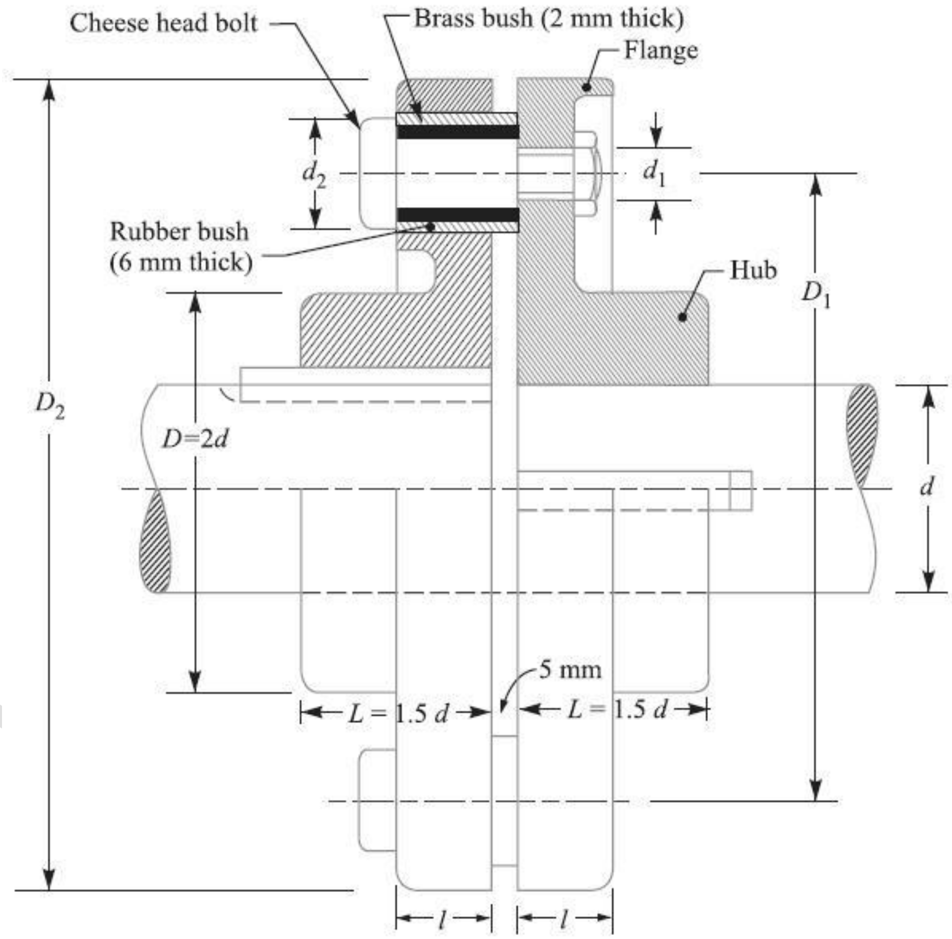
= Design Data hand Book - S MD Jalaludin.

Flexible Coupling:

We have already discussed that a flexible coupling is used to join the abutting e nds of shafts. when they are not in exact alig nment. In the case of a direct coupled drive from a prime mover to an electric generator, we should have four bearings at a compa ratively close distance. In such a case and in many others, as in a direct electric drive from an electric motor to a machine tool, a flexible coupling is used so as to permit an axial misalig nemnt of the shaft without undue absorption o f the power which the shaft are transmitting.

Bushed-pin Flexible Coupling

A bushed-pin flexible co upling, as shown in Fig., is a modification of th e rigid type of flange coupling. The coupling bolts are known as pins.



The rubber or leather bushes are used over the pins. The two halves of the coupling are dissimilar in construction. A clearance of 5 mm is left between the face of the two halves of the coupling. There is no rigid c onnection between them and the drive takes plac e through the medium of the compressible rub ber or leather bushes.

In designing the bushed-pin flexible coupling, the proportions of the rigid type flange coupling are modified. The main modification is to reduce the bearing pressure on the rubber or leather bushes and it should not exceed 0.5 N/mm2. In order to keep the low bearing pressure, the pitch circle diamete r and the pin size is increased. Let l = Length of bush in the flange,

D_2 = Diameter of bush,

P_b = Bearing pressure on the bush or pin,

n = Number of pins, and

D_1 = Diameter of pitch circle of the pins.

We know that bearing lo ad acting on each pin,

$W = p_b \times d_2 \times l$

Then, Total bearing load on the bush or pins

1. $W \times n = p_b \times d_2 \times l \times n$

And the torque transmitted by the coupling,

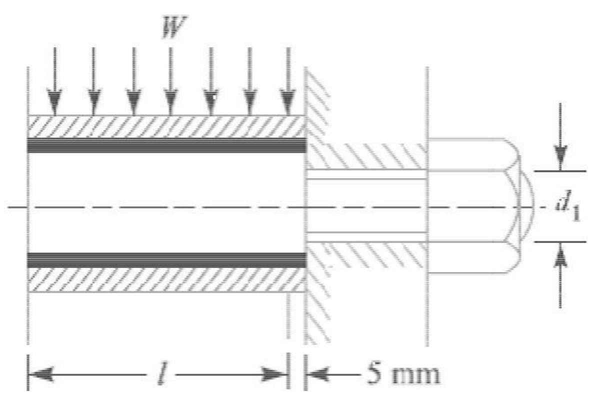
$$T = W \times n\left(\frac{D_1}{2}\right) = p_b \times d_2 \times l \times n\left(\frac{D_1}{2}\right)$$

The threaded portion of the pin in the right hand flange should be a tapping fit i n the coupling hole to avoid bending stresses.

The threaded length of the pin should be as small as possible so that the direc t shear stress can be taken by the unthreaded neck.

Direct shear stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2}$$



Since the pin and the rubber or leather bush is not rigidly held in the left hand flange, therefore the tangential load (W) at the enla rged portion will exert a bending action on the pin as shown in Fig. The bush portion of the pin acts as a cantilever beam of length l. Assuming a uniform distributi on of the load W along the bush, the maxi mum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5 \text{ mm}\right)$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{W\left(\frac{l}{2} + 5 \text{ mm}\right)}{\frac{\pi}{32}(d_1)^3}$$

Since the pin is subjected to bending and shear stresses, therefore the design must be checked either for the maximum principal stress or maximum shear stress by the following relations: Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

and the maximum shear stress on the pin

$$= \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

The value of maximum principal stress varies from 28 to 42 MPa.

Note: After designing the pins and rubber bush, the hub, key and flange may be designed in the similar way as discussed for flange coupling.

References:

= Machine Design - V.Bandari .

= Machine Design – R.S. Khurmi

= Design Data hand Book - S MD Jalaludin.

Problem:

Design a bushed-pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 kW at 960 r.p.m. The overall torque is 20 percent more than mean torque. The material properties are as follows:

1. The allowable shear and crushing stress for shaft and key material is 40 MPa and 80 MPa respectively.
2. The allowable shear stress for cast iron is 15 MPa.
3. The allowable bearing pressure for rubber bush is 0.8 N/mm2.
4. The material of the pin is same as that of shaft and key.

Draw neat sketch of the coupling.

Solution. Given: P = 32 kW = 32 × 10^3 W; N = 960 r.p.m. ; T_{max} = 1.2 T_{mean} ; $\tau_s = \tau_k$ = 40 MPa = 40 N/mm^2 ; $\sigma_{cs} = \sigma_{ck}$ = 80 MPa = 80 N/mm^2 ; τ_c = 15 MPa = 15 N/mm^2 ; p_b = 0.8 N/mm^2.

<u>1. Design for pins and rubber bush</u>

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960} = 318.3 \text{ N-m}$$

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 318.3 = 382 \text{ N-m} = 382 \times 10^3 \text{ N-mm}$$

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$d^3 = 382 \times 10^3 / 7.86 = 48.6 \times 10^3 \text{ or } d = 36.5 \text{ say } 40 \text{ mm}$$

$$d_1 = \frac{0.5\, d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{6}} = 8.2 \text{ mm}$$

In order to allow for the bending stress induced due to the compressibility of the rubber bush, the diameter of the pin (d_1) may be taken as 20 mm. Ans.

The length of the pin of least diameter i.e. d_1 = 20 mm is threaded and secured in the right hand coupling half by a standard nut and washer. The enlarged portion of the pin which is in the left hand coupling half is made of 24 mm diameter. On the enlarged portion, a brass bush of thickness 2 mm is pressed. A brass bush carries a rubber bush. Assume the thickness of rubber bush as 6 mm.

So, Overall diameter of rubber bush,

d_2 = 24 + 2 × 2 + 2 × 6 = 40 mm Ans.

and diameter of the pitch circle of the pins,

D_1 = 2 d + d_2 + 2 × 6 = 2 × 40 + 40 + 12 = 132 mm Ans.

Let l = Length of the bush in the flange.

We know that the bearing load acting on each pin,

$W = p_b \times d_2 \times l = 0.8 \times 40 \times l = 32\ l$ N

And the maximum torque transmitted by the coupling (T_{max}),

$$382 \times 10^3 = W \times n \times \frac{D_1}{2} - 32\,l \times 6 \times \frac{132}{2} = 12\,672\,l$$

l = 382 × 103/12 672 = 30.1 say 32 mm

And W = 32 l = 32 × 32 = 1024 N

So, Direct stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2} = \frac{1024}{\frac{\pi}{4}(20)^2} = 3.26 \text{ N/mm}^2$$

Since the pin and the rubber bush are not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin. Assuming a uniform distribution of load (W) along the bush, the maximum bending moment on the pin,

$$M = W\left(\frac{l}{2} + 5\right) = 1024\left(\frac{32}{2} + 5\right) = 21\,504 \text{ N-mm}$$

$$Z = \frac{\pi}{32}(d_1)^3 = \frac{\pi}{32}(20)^3 = 785.5 \text{ mm}^3$$

$$\sigma = \frac{M}{Z} = \frac{21\,504}{785.5} = 27.4 \text{ N/mm}^2$$

Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[27.4 + \sqrt{(27.4)^2 + 4\,(3.26)^2}\right]$$

$$= 13.7 + 14.1 = 27.8 \text{ N/mm}^2$$

And maximum shear stress

$$= \frac{1}{2}\left[\sqrt{\sigma^2 + 4\tau^2}\right] = \frac{1}{2}\left[\sqrt{(27.4)^2 + 4\,(3.26)^2}\right] = 14.1 \text{ N/mm}^2$$

Since the maximum principal stress and maximum shear stress are within limits, therefore the design is safe.

2. Design for hub

We know that the outer diameter of the hub,

D = 2 d = 2 × 40 = 80 mm

And length of hub, L = 1.5 d = 1.5 × 40 = 60 mm

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(80)^4 - (40)^4}{80}\right] = 94.26 \times 10^3 \tau_c$$

τ_c = 382 × 103 / 94.26 × 103 = 4.05 N/mm^2 = 4.05 MPa

Since the induced shear stress for the hub material (i.e. cast iron) is less than the permissible value of 15 MPa, therefore the design of hub is safe.

3. Design for key

Since the crushing stress for the key material is twice its shear stress (i.e. $\sigma_{ck} = 2\ \tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 40 mm diameter,

Width of key, w = 14 mm Ans.

and thickness of key,t = w = 14 mm Ans.

The length of key (L) is taken equal to the length of hub, i.e.

L = 1.5 d = 1.5 × 40 = 60 mm

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times w \times \tau_k \times \frac{d}{2} = 60 \times 14 \times \tau_k \times \frac{40}{2} = 16\,800\ \tau_k$$

$$\tau_k = 382 \times 10^3/16\,800 = 22.74 \text{ N/mm}^2 = 22.74 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = L \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 60 \times \frac{14}{2} \times \sigma_{ck} \times \frac{40}{2} = 8400\ \sigma_{ck}$$

σ_{ck} = 382 × 103/8400 = 45.48 N/mm2 = 45.48 MPa

Since the induced shear and crushing stress in the key are less than the permissible stresses of 40 MPa and 80 MPa respectively, therefore the design for key is safe.

4. Design for flange

The thickness of flange (t_f) is taken as 0.5 d.

t_f = 0.5 d = 0.5 × 40 = 20 mm

Let us now check the induced shear stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$382 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (80)^2}{2} \times \tau_c \times 20 = 201 \times 10^3 \tau_c$$

τ_c =382 × 103 / 201 × 103 = 1.9 N/mm2 = 1.9 MPa

Since the induced shear stress in the flange of cast iron is less than 15 MPa, therefore the design of flange is safe.

References:

1. Machine Design - V.Bandari .
2. Machine Design – R.S. Khurmi
3. Design Data hand Book - S MD Jalaludin.

Problem:

Design a cast iron protective ty pe flange coupling to transmit 15 kW at 900 r .p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. T he following permissible stresses may be used:

Shear stress for shaft, bolt and key material = 40

MPa Crushing stress for bolt and key = 80 MPa

Shear stress for cast iron = 8 MPa

Draw a neat sketch of the coupli ng.

Solution. Given: P = 15 kW = 15 × 103 W; N = 900 r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau_b = \tau_k$ = 40 MPa = 40 N/mm^2 ; $\sigma_{cb} = \sigma_{ck}$ = 80 MPa = 80 N/mm^2 ; τ_c = 8 MPa = 8 N/m m^2.

The protective type flange coupling is designed as discussed below:

1. Design for hub

First of all, let us find the diam eter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

T_{max} = 1.35 × 159.13 = 215 N-m = 215 × 103 N-mm

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3 \quad \text{or} \quad d = 30.1 \text{ say } 35 \text{ mm Ans.}$$

We know that outer diameter of the hub,

D = 2d = 2 × 35 = 70 mm Ans.

And length of hub, L = 1.5 d = 1.5 × 35 = 52.5 mm Ans.

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\,147\, \tau_c$$

Then, τ_c = 215 × 103/63 147 = 3.4 N/mm2 = 3.4 MPa

Since the induced shear stress fo r the hub material (i.e. cast iron) is less than th e permissible value of 8 MPa, therefore the de sign of hub is safe.

2. Design for key

Since the crushing stress for the key material is twice its shear stress (i.e. $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From DDB, we find that for a shaft of 35 mm diameter,

Width of key, w = 12 mm Ans.

And thickness of key, t = w = 12 mm Ans.

The length of key (l) is taken equal to the length of hub.

Then, l = L = 52.5 mm Ans.

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\,025\ \tau_k$$

Then, τ_k = 215 × 103/11 025 = 19.5 N/mm2 = 19.5 MPa

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

Σ_{ck} = 215 × 103/ 5512.5 = 39 N/mm^2 = 39 MPa.

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. Design for flange

The thickness of flange (t_f) is taken as 0.5 d.

Then, t_f = 0.5 d = 0.5 × 35 = 17.5 mm Ans.

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\,713\ \tau_c$$

τ_c = 215 × 103/134 713 = 1.6 N/mm2 = 1.6 MPa

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. Design for bolts

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

n = 3 and pitch circle diameter of bolts,

$D_1 = 3d = 3 \times 35 = 105$ mm

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4}(d_1)^2 \ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2 \ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$$(d_1)^2 = 215 \times 103/4950 = 43.43 \text{ or } d_1 = 6.6 \text{ mm}$$

Assuming coarse threads, the nearest standard size of bolt is M 8. Ans.

Other proportions of the flange are taken as follows:

Outer diameter of the flange,

$D_2 = 4\ d = 4 \times 35 = 140$ mm Ans.

Thickness of the protective circumferential flange,

$t_p = 0.25\ d = 0.25 \times 35 = 8.75$ say 10 mm Ans.

References:

1. Machine Design - V.Bandari .

2. Machine Design – R.S. Khurmi

3. Design Data hand Book - S MD Jalaludin.

Problem:

Two 35 mm shafts are connected by a flanged coupling. The flanges are fitted with 6 bolts on 125 mm bolt circle. The shafts transmit a torque of 800 N-m at 350 r.p.m. For the safe stresses mentioned below, calculate 1. Diameter of bolts; 2. Thickness of flanges; 3. Key dimensions ; 4. Hub length; and 5. Power transmitted. Safe shear stress for shaft material = 63 MPa Safe stress for bolt material = 56 MPa Safe stress for cast iron coupling = 10 MPa Safe stress for key material = 46 MPa

Solution. Given: d = 35 mm; n = 6; D1 = 125 mm; T = 800 N-m = 800 × 103 N-mm; N = 350 r.p.m.; τs = 63 MPa = 63 N/mm^2; τ_b = 56 MPa = 56 N/mm^2 ; τ_c = 10 MPa = 10 N/mm^2 ; τ_k = 46 MPa = 46 N/mm^2.

1. Diameter of bolts

Let d_1 = Nominal or outside diameter of bolt. We know that the torque transmitted (T),

$$800 \times 10^3 = \frac{\pi}{4}(d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2\, 56 \times 6 \times \frac{125}{2} = 16\,495\,(d_1)^2$$

$(d_1)^2 = 800 \times 10^3 / 16\,495 = 48.5$ or $d_1 = 6.96$ say 8 mm **Ans.**

2. Thickness of flanges

Let t_f = Thickness of flanges.

We know that the torque transmitted (T),

$$800 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi\,(2 \times 35)^2}{2} \times 10 \times t_f = 76\,980\,t_f \quad \ldots (\because D = 2d)$$

t_f = 800 × 103 / 76 980 = 10.4 say 12 mm **Ans.**

3. Key dimensions

From Table 13.1, we find that the proportions of key for a 35 mm diameter shaft are:

Width of key, w = 12 mm **Ans.**

And thickness of key, t = 8 mm**Ans.**

The length of key (l) is taken equal to the length of hub (L).

l = L = 1.5 d = 1.5 × 35 = 52.5 mm

Let us now check the induced shear stress in the key. We know that the torque transmitted (T),

$$800 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} \times = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\ 025\ \tau_k$$

τ_k = 800 × 103 / 11 025 = 72.5 N/mm2

Since the induced shear stress in the key is more than the given safe stress (46 MPa), therefore let us find the length o f key by substituting the value of τk = 46 MPa in the above equation, i.e.

$$800 \times 10^3 = l \times 12 \times 46 \times \frac{35}{2} = 9660\, l$$

l = 800 × 103 / 9660 = 82.8 say 85 mm **Ans.**

4. Hub length

Since the length of key is taken e qual to the length of hub, therefore we shall take hub length,

L = l = 85 mm **Ans.**

5. Power transmitted

We know that the power transmitted,

$$P = \frac{T \times 2\pi N}{60} = \frac{800 \times 2\pi \times 350}{60} = 29\ 325 \text{ W} = 29.325 \text{ kW Ans.}$$

Problem:

The shaft and the flange of a ma rine engine are to be designed for flange coup ling, in which the flange is forged on the end of the shaft. The following particulars are to considered in the design:

Power of the engine = 3 MW

Speed of the engine = 100 r.p.m.

Permissible shear stress in bolts and shaft = 60

MPa Number of bolts used = 8

Pitch circle diameter of bolts = 1.6 × Diameter of shaft

Find: 1. diameter of shaft; 2. diameter of bolts; 3. thickness of flange; and 4 . diameter of flange.

Solution. Given: P = 3 MW = 3 × 106 W; N = 100 r.p.m.; τb = τs = 60 MPa = 60 N/mm^2; n = 8; D1 = 1.6 d

1. Diameter of shaft

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{3 \times 10^6 \times 60}{2\pi \times 100} = 286 \times 10^3 \text{ N-m} = 286 \times 10^6 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$286 \times 10^6 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

$d_3 = 286 \times 10^6 / 11.78 = 24.3 \times 10^6$

or $d = 2.89 \times 10^2 = 289$ say 300 mm **Ans.**

2. Diameter of bolts

Let d_1 = Nominal diameter of bolts.

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$286 \times 10^6 = \frac{\pi}{4} (d_1)^2 \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} \times (d_1)^2\ 60 \times 8 \times \frac{1.6 \times 300}{2}$$

$= 90\,490\, (d_1)^2$ ... (Since $D_1 = 1.6$ d)

So, $(d_1)^2 = 286 \times 10^6 / 90\,490 = 3160$ or $d_1 = 56.2$ mm

Assuming coarse threads, the standard diameter of the bolt is 60 mm (M 60). The taper on the bolt may be taken from 1 in 20 to 1 in 40. **Ans.**

3. Thickness of flange

The thickness of flange (t_f) is taken as d / 3.

So, tf = d / 3 = 300/3 = 100 mm **Ans.**

Let us now check the induced shear stress in the flange by considering the flange at the junction of the shaft in shear. We know that the torque transmitted (T),

$$286 \times 10^6 = \frac{\pi d^2}{2} \times \tau_s \times t_f = \frac{\pi (300)^2}{2} \times \tau_s \times 100 = 14.14 \times 10^6\, \tau_s$$

$\tau_s = 286 \times 10^6 / 14.14 \times 10^6 = 20.2 \text{ N/mm}^2 = 20.2$ MPa

Since the induced shear stress in the *flange is less than the permissible shear stress of 60 MPa, therefore the thickness of flange (tf = 100 mm) is safe.

4. Diameter of flange

The diameter of flange (D_2) is taken as 2.2 d.

So, $D_2 = 2.2\, d = 2.2 \times 300 = 660$ mm **Ans.**

Cottered Joints:

A cotter is a flat wedge shaped piece of rectangular cross-section and its wi dth is tapered (either on one side or both sides) from one end to another for an easy adjustm ent. The taper varies from 1 in 48 to 1 in 24 and it may be increased up to 1 in 8, if a lock ing device is provided. The locking device may be a taper pin or a set screw used on the low er end of the cotter. The cotter is usually made of mild steel or wrought iron. A cotter joint is a temporary fastening and is used to connect rigidly two co-axial rods or bars which are subjjected to axial tensile or compressive forces. It is usually used in connecting a piston rod to the crosshead of a reciprocating steam engine, a piston rod and its extension as a tail or pump rod , strap end of connecting rod etc.

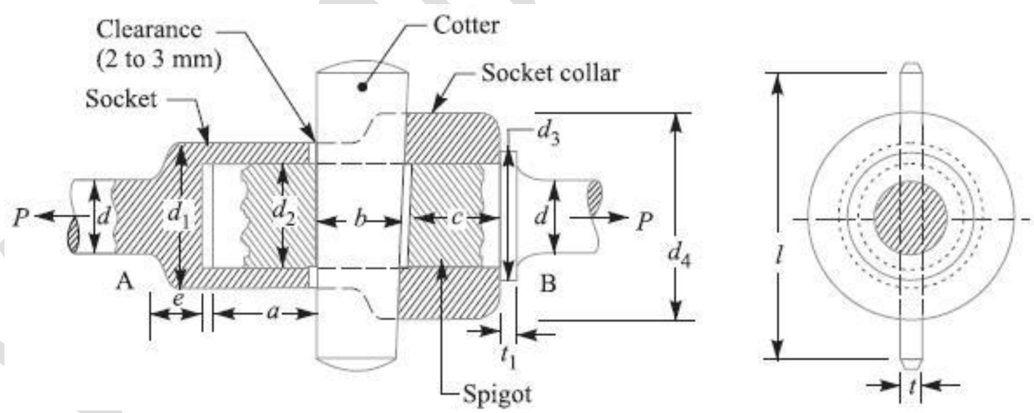
Types of Cotter Joints

Following are the three commonly used cotter joints to connect two rods by a cotter:

2. Socket and spigot cotter joint, **2.** Sleeve and cotter joint, and **3.** Gib and cotter joint.

Socket and Spigot Cotter Joint

In a socket and spigot cotter joinnt, one end of the rods (say *A*) is provided with a socket type of end as shown in Fig., and the other end of the other rod (say *B*) is inserted into a socket. The end of the rod which goes into a socket is also called ***spigot.*** *A* rectangular hole is made in the socket and spigot. *A* cotteer is then driven tightly through a hole in order to make the temporary connection between the two rods. The load is usually acting axially, but it changes its direction and hence the cotter joint must be designed to carry both the tensile and compressive loads. The compressive load is taken up by the collar on the spigot.



Fi g. Socket and spigot cotter joint

Design of Socket and Spigot C otter Joint

The socket and spigot cotter joint is shown in Fig.

Let P = Load carried by the rods,

d = Diameter of the rods,

d_1 = Outside diameter of socket,

d_2 = Diameter of spigot or inside diameter of

socket, d_3 = Outside diameter of spigot collar, t_1= Thickness of spigot collar,

d_4 = Diameter of socket collar,

c = Thickness of socket collar,

b = Mean width of cotter,

t = Thickness of cotter,

l = Length of cotter,

a = Distance from the en d of the slot to the end of rod,

σ_t = Permissible tensile stress for the rods material,

5. **=** Permissible shear stress for the cotter material, and

σ_c = Permissible crushing stress for the cotter material.

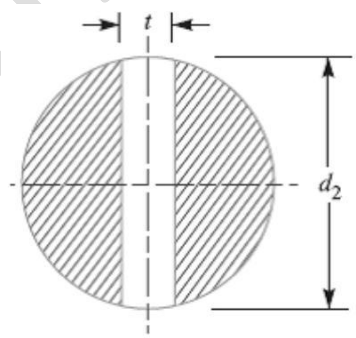
The dimensions for a socket and spigot cotter joint may be obtained by considering the various modes of failure as discussed below:

1. Failure of the rods in tension

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rods (d) may be determined.

2. Failure of spigot in tension across the weakest section (or slot)



$$P = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right] \sigma_t$$

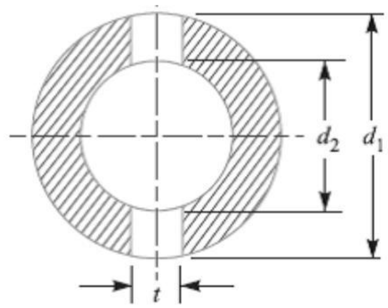
From this equation, the diameeter of spigot or inside diameter of socket (d_2) may be determined. In actual practice, the thickness of cotter is usually taken as d_2 / 4.

3. Failure of the rod or cotter in crushing

$$P = d_2 \times t \times \sigma_c$$

From this equation, the induced crushing stress may be checked.

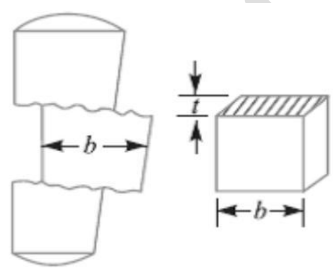
4. Failure of the socket in tension across the slot



$$P = \left\{\frac{\pi}{4}\left[(d_1)^2 - (d_2)^2\right] - (d_1 - d_2)\, t\right\} \sigma_t$$

From this equation, outside diameter of socket (d_1) may be determined.

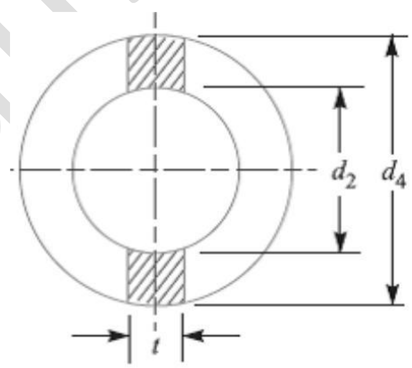
5. Failure of cotter in shear



$$P = 2\, b \times t \times \tau$$

From this equation, width of cotter (b) is determined.

6. Failure of the socket collar in crushing



$$P = (d_4 - d_2)\, t \times \sigma_c$$

From this equation, the diameter of socket collar (d_4) may be obtained.

7. Failure of socket end in shearing

$$P = 2\,(d_4 - d_2)\, c \times \tau$$

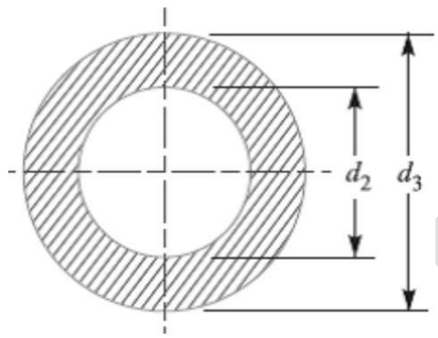
From this equation, the thicknesss of socket collar (c) may be obtained.

8. Failure of rod end in shear

$$P = 2\,a \times d_2 \times \tau$$

From this equation, the distance from the end of the slot to the end of the rod (a) may be obtained.

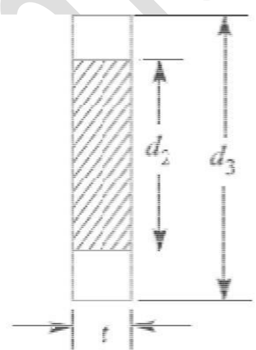
9. Failure of spigot collar in crushing



$$P = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c$$

From this equation, the diameter of the spigot collar (d_3) may be obtained.

10. Failure of the spigot collar in shearing

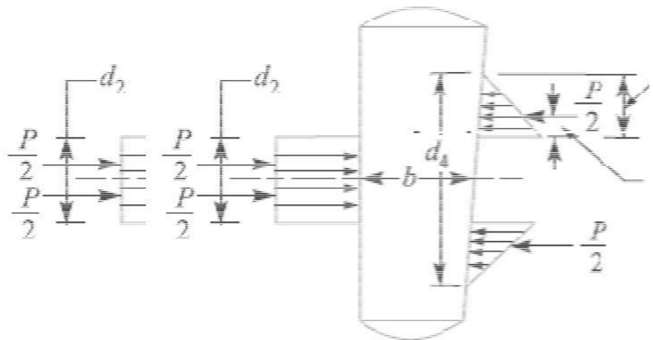


$$P = \pi\, d_2 \times t_1 \times \tau$$

From this equation, the thicknesss of spigot collar (t_1) may be obtained.

11. Failure of cotter in bending

The maximum bending moment occurs at the centre of the cotter and is given by



$$M_{max} = \frac{P}{2}\left(\frac{1}{3} \times \frac{d_4 - d_2}{2} + \frac{d_2}{2}\right) - \frac{P}{2} \times \frac{d_2}{4}$$

$$= \frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{2} - \frac{d_2}{4}\right) = \frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{4}\right)$$

We know that section modulus of the cotter,

$$Z = t \times b^2 / 6$$

Bending stress induced in the cotter,

$$\sigma_b = \frac{M_{max}}{Z} = \frac{\frac{P}{2}\left(\frac{d_4 - d_2}{6} + \frac{d_2}{4}\right)}{t \times b^2 / 6} = \frac{P\,(d_4 + 0.5\,d_2)}{2\,t \times b^2}$$

This bending stress induced in the cotter should be less than the allowable bending stress of the cotter.

5. The length of cotter (l) in taken as 4 d.

6. The taper in cotter should no t exceed 1 in 24. In case the greater taper is required, then a locking device must be provided.

7. The draw of cotter is generally taken as 2 to 3 mm.

Notes: 1. when all the parts of the joint are made of steel, the following proportions in terms of diameter of the rod (d) are gennerally adopted:

d_1 = 1.75 d , d_2 = 1.21 d , d_3 = 1.5 d , d_4 = 2.4 d , a = c = 0.75 d , b = 1.3 d, l = 4 d , t = 0.31 d ,t_1 = 0.45 d , e = 1.2 d.

Taper of cotter = 1 in 25, and dra w of cotter = 2 to 3 mm.

4. If the rod and cotter are made of steel or wrought iron, then $\tau = 0.8\ \sigma_t$ and $\sigma_c = 2\ \sigma_t$ may be taken.

Problem:

Design and draw a cotter joint to support a load varying from 30 kN in compression to 30 kN in tension. The material used is carbon steel for which the following allowable stresses may be used. The load is applied stattically. Tensile stress = compressive stress = 500 MPa ; shear stress = 35 MPa and crushing stress = 90 MPa.

Solution. Given : $P = 30\text{ kN} = 30 \times 10^3\text{ N}$; $\sigma_t = 50\text{ MPa} = 50\text{ N/mm}^2$; $\tau = 35\text{ MPa} = 35\text{ N/mm}^2$; $\sigma_c = 90\text{ MPa} = 90\text{ N/mm}^2$

1. *Diameter of the rods*

Let d = Diameter of the rods.

Considering the failure of the rod in tension. We know that load (P),

$$30 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 50 = 39.3\, d^2$$

$\therefore$ $d^2 = 30 \times 10^3 / 39.3 = 763$ or $d = 27.6$ say 28 mm **Ans.**

2. *Diameter of spigot and thickness of cotter*

Let d_2 = Diameter of spigot or inside diameter of socket, and

t = Thickness of cotter. It may be taken as $d_2 / 4$.

Considering the failure of spigot in tension across the weakest section. We know that load (P),

$$30 \times 10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times \frac{d_2}{4}\right]50 = 26.8\,(d_2)^2$$

$\therefore$ $(d_2)^2 = 30 \times 10^3 / 26.8 = 1119.4$ or $d_2 = 33.4$ say 34 mm

and thickness of cotter, $t = \frac{d_2}{4} = \frac{34}{4} = 8.5\text{ mm}$

Let us now check the induced crushing stress. We know that load (P),

$$30 \times 10^3 = d_2 \times t \times \sigma_c = 34 \times 8.5 \times \sigma_c = 289\,\sigma_c$$

$\therefore$ $\sigma_c = 30 \times 10^3 / 289 = 103.8\text{ N/mm}^2$

Since this value of σ_c is more than the given value of $\sigma_c = 90\text{ N/mm}^2$, therefore the dimensions $d_2 = 34$ mm and $t = 8.5$ mm are not safe. Now let us find the values of d_2 and t by substituting the value of $\sigma_c = 90\text{ N/mm}^2$ in the above expression, *i.e.*

$$30 \times 10^3 = d_2 \times \frac{d_2}{4} \times 90 = 22.5\,(d_2)^2$$

$\therefore$ $(d_2)^2 = 30 \times 10^3 / 22.5 = 1333$ or $d_2 = 36.5$ say 40 mm **Ans.**

and $t = d_2 / 4 = 40 / 4 = 10$ mm **Ans.**

3. *Outside diameter of socket*

Let d_1 = Outside diameter of socket.

Considering the failure of the socket in tension across the slot. We know that load (P),

$$30 \times 10^3 = \left[\frac{\pi}{4}\left\{(d_1)^2 - (d_2)^2\right\} - (d_1 - d_2)\,t\right]\sigma_t$$

$$= \left[\frac{\pi}{4}\left\{(d_1)^2 - (40)^2\right\} - (d_1 - 40)\,10\right]50$$

$$30 \times 10^3/50 = 0.7854\,(d_1)^2 - 1256.6 - 10\,d_1 + 400$$

or $(d_1)^2 - 12.7\,d_1 - 1854.6 = 0$

$$\therefore \quad d_1 = \frac{12.7 \pm \sqrt{(12.7)^2 + 4 \times 1854.6}}{2} = \frac{12.7 \pm 87.1}{2}$$

$= 49.9$ say 50 mm **Ans.** ...(Taking +ve sign)

4. *Width of cotter*

Let b = Width of cotter.

Considering the failure of the cotter in shear. Since the cotter is in double shear, therefore load (P),

$$30 \times 10^3 = 2\,b \times t \times \tau = 2\,b \times 10 \times 35 = 700\,b$$

$$\therefore \quad b = 30 \times 10^3 / 700 = 43 \text{ mm } \textbf{Ans.}$$

5. *Diameter of socket collar*

Let d_4 = Diameter of socket collar.

Considering the failure of the socket collar and cotter in crushing. We know that load (P),

$$30 \times 10^3 = (d_4 - d_2)\,t \times \sigma_c = (d_4 - 40)10 \times 90 = (d_4 - 40)\,900$$

$$\therefore \quad d_4 - 40 = 30 \times 10^3 / 900 = 33.3 \text{ or } d_4 = 33.3 + 40 = 73.3 \text{ say } 75 \text{ mm } \textbf{Ans.}$$

6. *Thickness of socket collar*

Let c = Thickness of socket collar.

Considering the failure of the socket end in shearing. Since the socket end is in double shear, therefore load (P),

$$30 \times 10^3 = 2(d_4 - d_2)\,c \times \tau = 2\,(75 - 40)\,c \times 35 = 2450\,c$$

$$\therefore \quad c = 30 \times 10^3 / 2450 = 12 \text{ mm } \textbf{Ans.}$$

7. *Distance from the end of the slot to the end of the rod*

Let a = Distance from the end of slot to the end of the rod.

Considering the failure of the rod end in shear. Since the rod end is in double shear, therefore load (P),

$$30 \times 10^3 = 2\,a \times d_2 \times \tau = 2a \times 40 \times 35 = 2800\,a$$

$$\therefore \quad a = 30 \times 10^3 / 2800 = 10.7 \text{ say } 11 \text{ mm } \textbf{Ans.}$$

8. *Diameter of spigot collar*

Let d_3 = Diameter of spigot collar.

Considering the failure of spigot collar in crushing. We know that load (P),

$$30 \times 10^3 = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c = \frac{\pi}{4}\left[(d_3)^2 - (40)^2\right]90$$

or
$$(d_3)^2 - (40)^2 = \frac{30 \times 10^3 \times 4}{90 \times \pi} = 424$$

$$\therefore \quad (d_3)^2 = 424 + (40)^2 = 2024 \text{ or } d_3 = 45 \text{ mm } \textbf{Ans.}$$

9. *Thickness of spigot collar*

Let t_1 = Thickness of spigot collar.

Considering the failure of spigot collar in shearing. We know that load (P),

$$30 \times 10^3 = \pi\,d_2 \times t_1 \times \tau = \pi \times 40 \times t_1 \times 35 = 4400\,t_1$$

$$\therefore \quad t_1 = 30 \times 10^3 / 4400 = 6.8 \text{ say } 8 \text{ mm } \textbf{Ans.}$$

10. The length of cotter (l) is taken as $4\,d$.

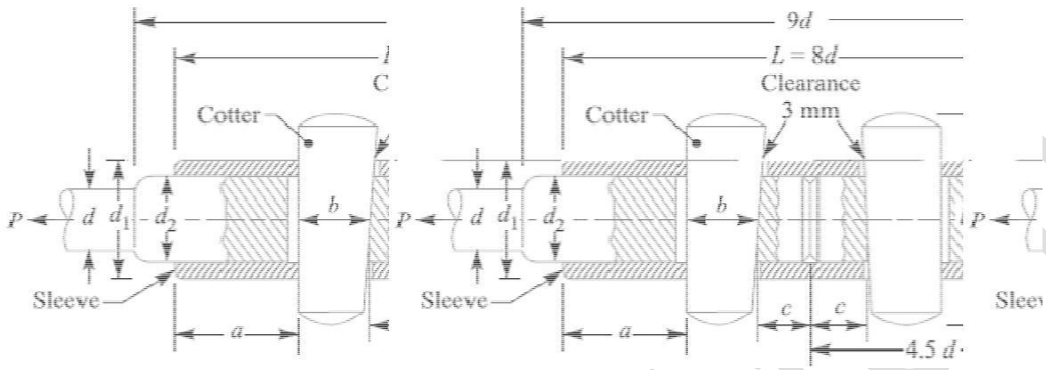
$$\therefore \quad l = 4\,d = 4 \times 28 = 112 \text{ mm } \textbf{Ans.}$$

11. The dimension e is taken as $1.2\,d$.

$$\therefore \quad e = 1.2 \times 28 = 33.6 \text{ say } 34 \text{ mm } \textbf{Ans.}$$

Sleeve and Cotter Joint

Sometimes, a sleeve and cotter joint as shown in Fig., is used to connect two round rods or bars. In this type of joint, a sleeve or muff is used over the two rods and then two cotters (one on each rod end) are inserted in the holes provided for them in the sleeve and roods. The taper of cotter is usually 1 in 24. It maay be noted that the taper sides of the two cotters should face each other as shown in Fig. The clearance is so adjusted that when the cotters are driven in, the two rods come closer to each other thus making the joint tight.



The various proportions for the sleeve and cotter joint in terms of the diameter of rod (d) are as follows :

Outside diameter of sleeve,

d_1 = 2.5 d

Diameter of enlarged end of rod,

d_2 = Inside diameter of sleeve = 1.25 d

Length of sleeve, L = 8 d

Thickness of cotter, t = d2/4 or 0.31 d

Width of cotter, b = 1.25 d

Length of cotter, l = 4 d

Distance of the rod end (a) fro m the beginning to the cotter hole (inside the sleeve end) = Distance of the rod end (c) from its end to the cotter hole = 1.25 d

Design of Sleeve and Cotter Joint

The sleeve and cotter joint is shown in Fig.

Let P = Load carried by the rods,

d = Diameter of the rods,

d_1 = Outside diameter of sleeve,

d_2 = Diameter of the enlarged end of rod,

t = Thickness of cotter,

l = Length of cotter,

b = Width of cotter,

a = Distance of the rod end from the beginning to the cotter hole (inside the sleeve end),

c = Distance of the rod end from its end to the cotter hole,

σ_t , τ and σ_c = Permissible tensile, shear and crushing stresses respectively for the material of the rods and cotter.

The dimensions for a sleeve and cotter joint may be obtained by considering the various modes of failure as discussed below:

1. Failure of the rods in tension

The rods may fail in tension due to the tensile load P. We know that

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rods (d) may be obtained.

2. Failure of the rod in tension across the weakest section (i.e. slot)

$$P = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t$$

From this equation, the diameter of enlarged end of the rod (d_2) may be obtained. The thickness of cotter is usually taken as d_2 / 4.

3. Failure of the rod or cotter in crushing

$$P = d_2 \times t \times \sigma_c$$

From this equation, the induced crushing stress may be checked.

4. Failure of sleeve in tension across the slot

$$P = \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\, t\right]\sigma_t$$

From this equation, the outside diameter of sleeve (d_1) may be obtained.

5. Failure of cotter in shear

$$P = 2b \times t \times \tau$$

From this equation, width of cotter (b) may be determined.

6. Failure of rod end in shear

$$P = 2\,a \times d_2 \times \tau$$

From this equation, distance (a) may be determined.

7. Failure of sleeve end in shear

$$P = 2\,(d_1 - d_2)\,c \times \tau$$

From this equation, distance (c) may be determined.

Problem:

Design a sleeve and cotter joint to resist a tensile load of 60 kN. All parts of the joint are made of the same material with the following allowable stresses: $\boldsymbol{\sigma_t}$ = 60 MPa ; $\boldsymbol{\tau}$ = 70 MPa ; and $\boldsymbol{\sigma_c}$ = 125 MPa.

Solution. Given : $P = 60$ kN $= 60 \times 10^3$ N ; $\sigma_t = 60$ MPa $= 60$ N/mm^2 ; $\tau = 70$ MPa $= 70$ N/mm^2 ; $\sigma_c = 125$ MPa $= 125$ N/mm^2

1. *Diameter of the rods*

Let d = Diameter of the rods.

Considering the failure of the rods in tension. We know that load (P),

$$60 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 60 = 47.13\, d^2$$

$$\therefore \quad d^2 = 60 \times 10^3 / 47.13 = 1273 \quad \text{or} \quad d = 35.7 \text{ say } 36 \text{ mm Ans.}$$

2. *Diameter of enlarged end of rod and thickness of cotter*

Let d_2 = Diameter of enlarged end of rod, and

t = Thickness of cotter. It may be taken as $d_2 / 4$.

Considering the failure of the rod in tension across the weakest section (*i.e.* slot). We know that load (P),

$$60 \times 10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times \frac{d_2}{4}\right]60 = 32.13\,(d_2)^2$$

$$\therefore \quad (d_2)^2 = 60 \times 10^3 / 32.13 = 1867 \quad \text{or} \quad d_2 = 43.2 \text{ say } 44 \text{ mm Ans.}$$

and thickness of cotter,

$$t = \frac{d_2}{4} = \frac{44}{4} = 11 \text{ mm Ans.}$$

Let us now check the induced crushing stress in the rod or cotter. We know that load (P),

$$60 \times 10^3 = d_2 \times t \times \sigma_c = 44 \times 11 \times \sigma_c = 484\,\sigma_c$$

$$\therefore \quad \sigma_c = 60 \times 10^3 / 484 = 124 \text{ N/mm}^2$$

Since the induced crushing stress is less than the given value of 125 N/mm^2, therefore the dimensions d_2 and t are within safe limits.

3. *Outside diameter of sleeve*

Let d_1 = Outside diameter of sleeve.

Considering the failure of sleeve in tension across the slot. We know that load (P)

$$60 \times 10^3 = \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\,t\right]\sigma_t$$

$$= \left[\frac{\pi}{4}[(d_1)^2 - (44)^2] - (d_1 - 44)\,11\right]60$$